CALCULATION OF THE EFFICIENCY OF POROUS CYLINDRICAL CHANNELS WITH A TURBULENT FLOW OF A LIQUID COOLANT UNDER THE BOUNDARY CONDITIONS OF THE FIRST TYPE

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Abstract

An important problem of the present time is the manufacturing of unobstructive highly efficient heat exchangers. One of the methods of enhancement for heat exchangers is application of porous materials with a high heat conductivity. Along with the obvious advantage – the high efficiency of heat transfer due to the high heat conductivity of porous insert material, there is also a disadvantage – high hydraulic resistance of porous structures. The purpose of this work was to reveal parameter of porous heat exchangers, when the gain in heat transfer would exceed the losses in hydraulics. Computation of comparative efficiency was carried out for porous materials made from metal-felt with a turbulent motion of an incompressible fluid – water – and first-type boundary condition. A smooth-wall tube was taken as a reference surface for comparison. Calculations showed that the positive effect is reached at a small diameter of the channel - about 3 mm, the value of Reynolds numbers of a turbulent flow in a smooth channel - Re_0 = 10 000–20 000, values of porosity of about 0.8–0.9 and the relative length of the channel of about 20.

KEYWORDS

Porous materials, coefficients of effectiveness, turbulent motion of incompressible fluid- water, first-type boundary condition.

INTRODUCTION

The important problem of the present time is the fabrication of unobstructive highly efficient heat exchangers. Porous heat-transmitting elements, prepared from metal-felt, metal-powder, highly porous cellular materials or from mesh materials can serve as examples of this type of systems. As a rule, copper or another analogous material is selected as the material of porous structure because of its high coefficient of thermal conductivity.

However, together with the explicit advantage of porous heat exchangers – the high efficiency of heat transfer due to the high thermal conductivity of the material of a porous insert – there is also a deficiency – high hydraulic resistance of porous structures. In spite of the sufficiently widespread investigation of hydraulics and heat exchange in porous materials, the energy effectiveness of these structures in comparison with the traditional smooth-wall channels had been given insufficient attention. This work is devoted to the study of this question in connection with turbulent motion of single-phase liquid flows and heating under the first-type boundary condition.

FORMULATION OF THE PROBLEM

The calculation of the effectiveness of porous heat exchangers was conducted using the methodology of Guhman, presented in [1]. This methodology compares three parameters: the quantity of transferred heat Q, the power consumed for heat carrier pumping N, and the area of the lateral surface of channels F. At the same time, any two of three parameters mentioned above are considered to be constant, and comparison is performed basing on the third one. Therefore, there can be three coefficients of effectiveness ratio: $k_Q = Q_p/Q_{sm}$ (thermal coefficient); $k_N = N_p/N_{sm}$ (power coefficient), and $k_F = F_p/F_{sm}$ (surface or dimensional coefficient). Since a smooth-wall tube was taken as a reference surface for comparison, the indices of the three coefficients given above represent: p – porous and sm

– smooth-wall cylindrical channel. If the diameters of channels are identical, $k_F = k_\xi = (\xi_p/\xi_{sm})$, where $\xi = x/d$ is the dimensionless length of channel, x the coordinate along the axis of the channel, and d is the diameter of the channel.

The calculation of the coefficients of effectiveness was carried out for a turbulent motion of an incompressible fluid - water - and first-type boundary condition. The quantity of heat, transmitted by the channel with a smooth wall, was calculated with the use of the average coefficient of convective heat exchange over the length of the channel .This coefficient was calculated from the formula of Mikheev [2].

The quantity of heat transferred by porous channels was conducted with the use of the average temperature of a fluid over the cross-section of a channel at an exit of the channel. The expression for the average temperature of a liquid over a cross-section of a porous channel under the first-type boundary condition was derived by Maiorov [3]. The relationships of Blasius and Nikuradse were used for calculation of hydraulic resistance in smooth-wall channels, and the modified equation of Darcy was used for porous channels. The resulting system of equations for calculating the coefficients k_Q, k_N, and k_F is written as follows:

$$\frac{\mu c_p}{4d}\cdot \mathrm{Re}_p \cdot [1-4\cdot\sum_{i=1}^{\infty}\frac{1}{\mu_n^2}\exp(-4\mu_n^2\cdot\xi p/\mathrm{Pe}_p(1+4\mu_n^2/\gamma^2))]\cdot(T_w-To)=\alpha_i\cdot\xi_o\cdot(Tw-\overline{T}_i) \quad (1)$$

$$\mathrm{Re}_p^3+\frac{\alpha d}{\beta}\cdot\mathrm{Re}_p^2-\frac{\xi'_{ot}}{2d\beta}\cdot\mathrm{Re}_o^3\cdot\frac{\xi_o}{\xi_p}=0 \quad (2)$$

at $\gamma^2 \le 10^3$, and

$$\frac{\mu c_p}{4d} Re_p[1-4\sum_{i=1}^{\infty}\frac{1}{\mu_n^2}exp(-B'_n\xi_p)]=\alpha_i\cdot\xi_o\cdot(Tw-\overline{T}_i), \quad (3)$$

where $B'_n=[(Pe/2)^2+4\mu_n^2]^{1/2}-Pe/2$

$$Re_p^3+\frac{\alpha d}{\beta}\cdot Re_p^2-\frac{\xi'_{ot}}{2d\beta}\cdot Re_o^3\cdot\frac{\xi_o}{\xi_p}=0 \quad (4)$$

at $\gamma^2 > 10^3$

Combined equations (1) – (2) are written for $\gamma^2 \le 10^3$, and combined equations (3)–(4) are written for $\gamma^2>10^3$.

Here $\gamma^2 = (h_v\cdot d^2)/\lambda_p$ is the parameter which characterizes the intensity of heat exchange inside of a porous material; h_v is the the intensity of volumetric heat exchange inside of a porous material; λ_p is the coefficient of the thermal conductivity of a porous material; $\overline{T}_i$ and α_i are the average temperature of a liquid over a length of a porous channel and the coefficient of convective heat exchange gained at the i-th step of iteration at calculation with the help of a method of successive approximationses and with use of the formula of Mikheev, ξ'_{ot} is the coefficient of hydraulic resistance in smooth-wall channels with turbulent motion of a single-phase liquid flow.

In Eqs. (1) and (3) the following designations are accepted: Re_p and Re_{sm} are the Reynolds numbers in porous and smooth-wall channels; $Pe_p = Re_p\cdot Pr_p = (G\cdot d\cdot c_p)/\lambda_p$ is the Peklets number of a porous channel; Pr_p is the Prandtl number of a porous channel; $G=\dot{m}/F_{cs}$ is the specific mass rate of a coolant flux; F_{cs} the cross-sectional area; $\dot{m}$ and c_p are the rate of the flux and heat capacity of the fluid; μ_n is the sequential roots of the equation $I_0(\mu) = 0$, (n = 1, 2, 3 ..., (μ_1 = 2,4048)), I_0 is the Bessel function of the first kind, zero degree. In this case, the expression in the brackets on the left side of equations (1) and (3) is the relative value of overshooting of the average temperature of the liquid relative to the temperature at the entrance into the porous channel:

$$1-\overline{\vartheta} = (\overline{t}-t_0)/(t_w-t_0) \quad (5)$$

Here, $\bar{t}$ is the average temperature of a liquid at the exit from the porous channels; the subscripts «w» and «0» refer to the temperature of the liquid on the wall and at the channel inlet, respectively.

In Eqs. (2) and (4), the parameters α and β designate the viscous and inertial coefficients of the resistance of a porous material.

The calculation of the parameter h_v in the expression for γ^2 is performed from the dimensionless equations of the form:

$$Nu = a \cdot Re^b \cdot Pr^c , \qquad (6)$$

where the coefficients a, b, and c are taken from the experimental data for a particular type of a porous material and heat transfer medium .

The Nusselt number in this equation is calculated using the formula $Nu = (h_v \cdot (\beta/\alpha)^2)/\lambda_l$, and Reynolds number is calculated from the relationship $Re = (G \cdot (\beta/\alpha))/\mu$, where λ_l is the thermal conductivity of heat-carrier agent, and μ is the coefficient of dynamic viscosity.

The system of equations obtained is the system of nonlinear algebraic equations with the variable coefficients. The solution of the problem in such statement setting is reduced to the search for a combination of the parameters of the porous structure and smooth-wall channel: porosity θ, diameter of the channels d, relative length of the smooth-wall channel $\xi_{sm} = x/d$, temperature of the wall of the channels (T_w) and the Reynolds number in the smooth-wall channel Re_{sm}, with which the obtained system of equations (1) – (2) or (3) – (4) has the best solution - the maximum value of the coefficients k_Q, k_N, and k_F .

SOLUTION

The calculations of the effectiveness coefficients were performed for the metal-felt, prepared from the fibers of copper with a diameter of 200 μm for the following calculated parameters: porosity θ = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, Reynolds number of the smooth channel: $Re_{sm} = 1 \cdot 10^4, 2 \cdot 10^4, 5 \cdot 10^4, 1 \cdot 10^5, 1 \cdot 10^6$; the relative length of the smooth-wall channel $\xi_{sm} = x/d$ = 2, 5, 20, 50, 100, 500, 1000; the diameter of a channel d = 1, 2, 3, 4, 5, 10, 20, 50 mm; the temperature of the wall T_w = 25, 30, 40, 70, 100 °C; the temperature of a liquid at the channel inlet T_0 = 20 °C.

The calculation of the coefficient of thermal conductivity of the porous material λ_p used the relationship obtained in [4] that agreed with the experimental data according to [5]. The calculation of the intensity of interstitial heat exchange was performed with the aid of the dimensionless equation [3] obtained experimentally for the porous material prepared from fibers:

$$Nu = 0{,}007 Re^{1{,}2} \qquad (7)$$

For calculating the parameters α and β the following relationships [5] were used:

$$\alpha = 2.57 \cdot 10^8 \cdot \theta^{-3{,}91} \qquad (8)$$

$$\beta = 0.91 \cdot 10^3 \cdot \theta^{-5{,}33}. \qquad (9)$$

In calculation of the coefficient of convective heat exchange under from the formula of Mikheev (10), as the determining temperature the average temperature over the length of the channel was taken. This temperature was determined by the method of successive approximationses from the formula

$$\overline{Nu}_{ж} = 0{,}021 \cdot Re_{ж}^{0{,}8} \cdot Pr_{ж}^{0{,}43} \cdot \left(\frac{Pr_{ж}}{Pr_c}\right)^{0{,}25} \cdot \varepsilon_l \qquad (10)$$

In the beginning of calculations, as the determining temperature the average temperature of the fluid $\overline{T_1}$ between the wall temperature T_w and the fluid temperature at the inlet T_0 was taken. Next, the coefficient of convective heat exchange is calculated from the formula (10) and the quantity of heat transferred by the channel with a smooth wall from formula (11):

$$Q_\alpha = \alpha \cdot (Tw - \overline{T_1}) \cdot F \ , \qquad (11)$$

where F is the cross-sectional area.

There after relation (11) was equated to the expression of the quantity of heat calculated from formula (12):

$$Q_{\bar{t}} = \dot{m} \cdot Cp \cdot (\overline{T}ex - To) \tag{12}$$

and the value of $\overline{T_{ex}}$ was determined. The average temperature of the fluid in the channel was further calculated from the relation $\overline{T_2} = \dfrac{T_0 + \overline{T}_{ex}}{2}$ and the iteration was made again as long as the successive approximations of the value of average temperature did not differ by the value Δ≈15 %. The calculation of the thermophysical properties of the fluid at this temperature was made from the done under an of Lagrange interpolation formula with the use of 10 base points in the temperatures interval of 0–100 °C. The correction factor ε_l in formula (10) takes into account the change in the average coefficient of convective heat exchange over the length of the channel.

In calculation of the coefficient of hydraulic resistance of a pipe with a smooth wall at $10^4 \le Re_d \le 10^5$ the formula of Blasius (13) was used:

$$\xi'_{ot} = 0{,}3164 \cdot Re_d^{-1/4}, \tag{13}$$

and at $10^5 < Re_d \le 10^6$ the formula of Nikuradse (14) was used:

$$\xi'_{ot} = 0{,}0032 + 0{,}0221 \cdot Re_d^{-0{,}237}. \tag{14}$$

In relations (13) and (14) Re_d is the Reynolds number based on the diameter of the channel.

Calculations were carried out for the entire field of parameters composed of 9800 points.

Coefficient k_Q. In calculation of the coefficient k_Q, using the requirements that $N_p = N_{sm}$ and $\xi_p = \xi_{sm}$, we gain and solve the Cardano cubic equation (2) or (4) for the variable Re_{sm}. Further, we substitute the obtained value of the variable Re_p and known value $\xi_p = \xi_{sm}$ in Eq. (1) or (3) and we calculate the coefficient k_Q, as the ratio of the left-hand side of the equation to its right side. Thus, on substitution of the values α_i and $\overline{T_i}$ d into the given equations, we use a method of iterations, as mentioned above. Table 1 presents a fragment of calculations of the coefficient k_Q for the parameters T_w = 25 °C, d = 0.003м, and ξ_{sm} = x/d = 20.

Table 1

Θ	Re_p				
	$1 \cdot 10^4$	$2 \cdot 10^4$	$5 \cdot 10^4$	10^5	10
0.3	0.161	0.198	0.239	0.270	0.321
0.4	0.281	0.340	0.405	0.450	0.427
0.5	0.426	0.510	0.600	0.647	0.497
0.6	0.596	0.707	0.804	0.811	0.518
0.7	0.788	0.916	0.956	0.884	0.504
0.8	0.983	1.065	0.975	0.857	0.452
0.9	1.046	0.988	0.823	0.687	0.347

$k_Q = f(Rep, \theta)$ T_0 = 20 °C; T_w = 25 °C; x/d = 20; d = 0.003 m

Coefficient k_N. In calculation of this coefficient, taking into account that k_F =1, i. e., $\xi_p = \xi_{sm}$, we solve the nonlinear algebraic equation (1) for the quantity Re_p by sorting out this variable from 0 up to

Re_{sm}. After finding the variable Re_p at which the condition $Q_p = Q_{sm}$ is satisfied and after substitution of this value in to equation (2), can be easily calculated the coefficient k_N:

$$k_N = \frac{2d\beta \cdot (Re_p^3 + \frac{\alpha d}{\beta} Re_p^2)}{\xi'_{ot} \cdot Re_{sm}^3} \tag{15}$$

Table 2 presents a fragment of calculations of the coefficient k_N for the parameters T_w=25^{0}C, d=0.003м, and ξ_{sm}=x/d=20.

Table 2

Θ	Re_p				
	$1\cdot10^4$	$2\cdot10^4$	$5\cdot10^4$	10^5	10^6
0.3	0.008	0.011	0.015	0.019	0.005
0.4	0.034	0.049	0.070	0.081	0.013
0.5	0.104	0.154	0.216	0.234	0.022
0.6	0.257	0.384	0.506	0.454	0.025
0.7	0.541	0.794	0.876	0.594	0.020
0.8	0.979	1.283	0.934	0.448	0.010
0.9	1.210	0.990	0.359	0.127	0.004

$k_N = f(Re_p, \theta)$, T_0 = 20 °C, T_w = 25 °C, x/d = 20, d = 0,003 m

Coefficient k_F. In calculation of the coefficient k_F, first it is necessary to express ξ_p and γ^2 from Eqs. (2) and (7) as function of Re_p :

$$\xi_p = \frac{\xi'_{ot}}{2d\beta} \cdot Re_{sm} \cdot \xi_{sm} \cdot \left(\frac{1}{Re_p^3 + \frac{\alpha d}{\beta} \cdot Re_p^2} \right) \tag{16}$$

$$\gamma^2 = 0,07 \cdot Re_p^{1,2} \cdot \left(\frac{\lambda_l}{\lambda_p}\right) \cdot \left(\frac{d}{\beta\alpha}\right)^{0,8} \tag{17}$$

and to substitute them into the left-hand side of Eq. (1). As a result, we shall obtain the nonlinear algebraic equation for the variable Re_p. This equation can be easily solved numerically by sorting out the values of R from 0 up to Re_{sm} or in the inverse order. After the determination of the value of Re_p from Eq. (2), it is possible to find the value of k_F :

$$k_F = \frac{\xi_{sm}}{\xi_p} = \frac{2d\beta \cdot (Re_p^3 + \frac{\alpha d}{\beta} Re_p^2)}{\xi'_{ot} \cdot Re_{sm}^3} \tag{18}$$

The data of calculation of magnitude k_F are submitted in the table 3.

Table 3

Θ	Re_p				
	$1\cdot10^4$	$2\cdot10^4$	$5\cdot10^4$	10^5	10^6
0.6	0.0	0.0	0.0	0.0	0.0
0.7	0.0	1.993	0.0	0.0	0.0
0.8	3.569	2.280	0.902	0.0	0.0
0.9	1.739	0.986	0.000	0.0	0.0

$k_F = f(Re_p, \theta)$ T_0 = 20 °C, T_w = 25 °C, x/d = 20, d = 0.003 m

The analysis of the calculations of the effectiveness coefficients of porous structures k_Q k_N, and k_F showed a general behavioral tendency with a change in the model’s parameters. The values of the indicated coefficients vary inversely proportionally to the change in the diameter of the channel d, the difference of temperatures $t_w – t_0$, the length of the compared smooth-wall channels ξ_{sm}, the porosity θ, and in the Reynolds number of the compared smooth-wall channels $Re_{sm.}$ Figures 1, 2, 3, 4 and 5 present the graphs of a change in the coefficients k_Q and k_N, depending on the diameter of a channel, the temperature of the channel wall, the Reynolds number and the dimensionless length of the compared smooth-wall channel, the porosity. The behaviour of the coefficient k_F depending on the diameter of the channel, Reynolds number and dimensionless length of the compared channel with a smooth wall is presented in Fig. 6, 7 and 8.

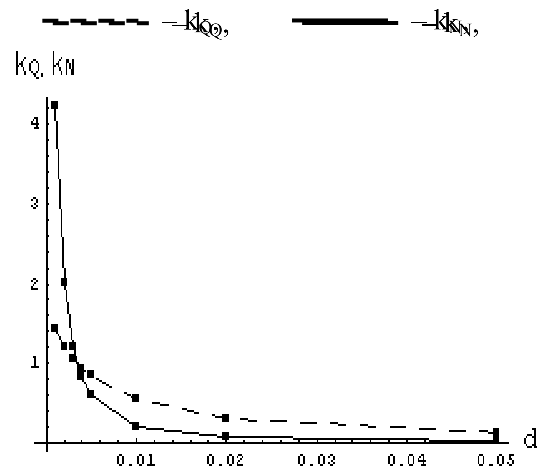


Fig. 1. Dependence of the coefficients k_Q and k_N on the diameter of the channel, k_Q, $k_N = f(d)$, $Re_{sm} = 10^4$; $\xi_{sm} = 20$; $T_0 = 20$ °C; $T_w = 25$ °C; $\theta = 0.9$

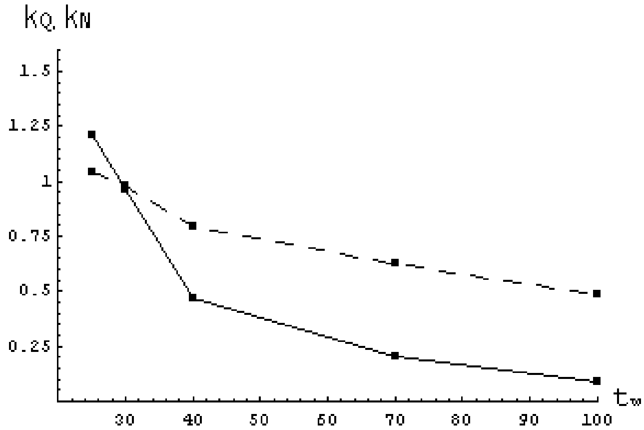
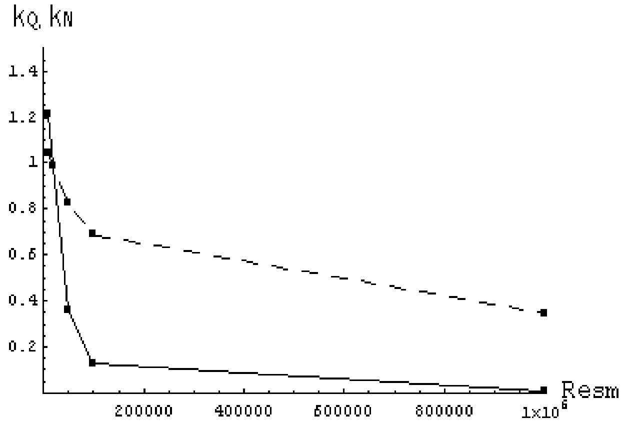


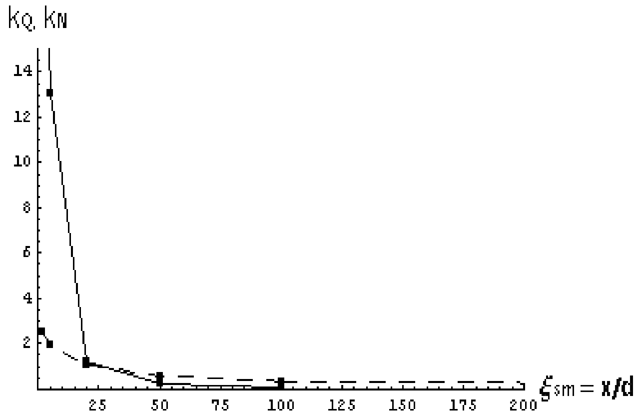
Fig. 2. Dependence of the coefficients k_Q and k_N on the temperature of the wall, $k_Q;k_N = f(t_w)$: $Re_{sm} = 10^4$, $\xi_{sm} = 20$, $T_0 = 20$ °C; $\theta = 0.9$, d = 3 mm



$\xi_{sm}=20$; $T_0=20^0C$; $Tw=25^0C$; d=3mm; $\theta=0{,}9$

- k_Q; - k_N

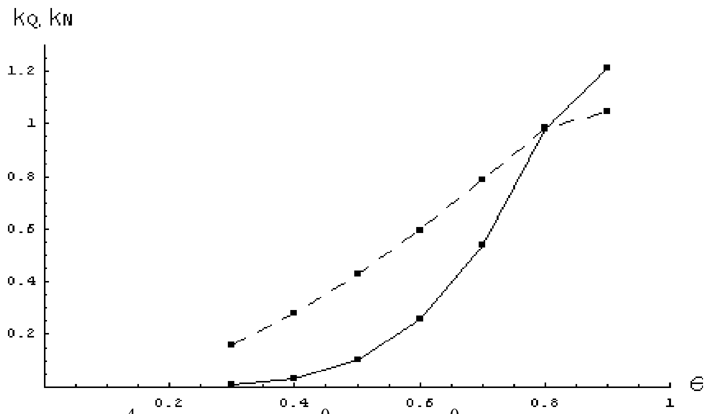
Fig. 3. Dependence of the coefficients k_Q and k_N on the Reynolds number of the channel with a smooth wall. $k_Q;k_N = f(Re_{sm})$.

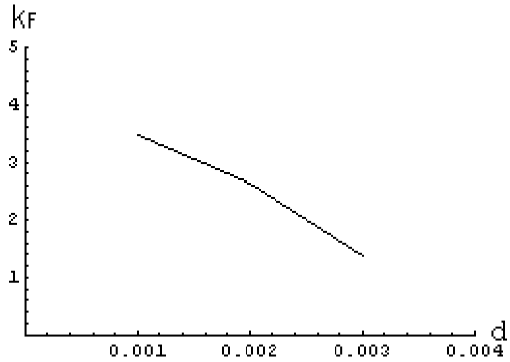


$Re_{sm}=10^4$; $T_0=20^0C$; $Tw=25^0C$; d=3mm; $\theta=0{,}9$

- k_Q; - k_N

Fig. 4. Dependence of the coefficients k_Q and k_N on the dimensionless length of the channel $k_Q;k_N = f(\xi_{sm})$



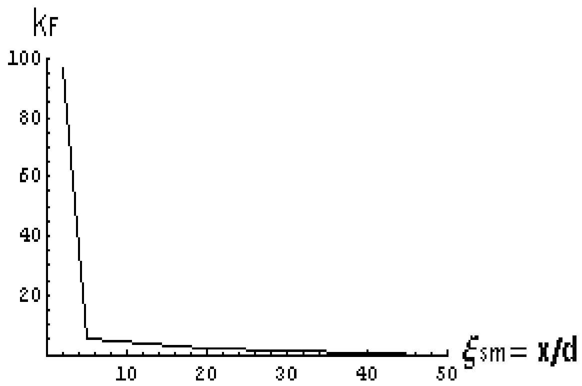


$Re_{sm}=10^4$; $\xi_{sm}=20$; $T_0=20^0C$; $Tw=25^0C$; d=3mm; $Re_{sm}=10^4$;$\xi_{sm}=20$;$T_0=20^0C$;$Tw=25^0C$;$\theta=0,9$

- - - - - k_Q; ——— - k_N

Fig. 5. Dependence of the coefficients k_Q and k_N on the porosity of the channel k_Q;$k_N = f(\theta)$

Fig. 6. Dependence of the coefficient k_F on the diameter of the channel $k_F = f(d)$



$\xi_{sm}=20$; $T_0=20^0C$; $Tw=25^0C$; d=3mm ;$\theta=0,9$ $Re_{sm}=10^4$;$T_0=20^0C$;$Tw=25^0C$; d=3mm; $\theta=0,9$

- - - - - k_Q; ——— - k_N

Fig. 5. Dependence of coefficient k_F from Reynolds number of a channel with smooth wall $k_F = f(Re_{sm})$

Fig. 6. Dependence of coefficient k_F from dimensionless length of a channel. $k_F = f(\xi_{sm})$

The following points were the initial parameters for the graphs: porosity $\theta = 0.9$; Reynolds number of the smooth-wall channel: $Re_{sm} = 10^4$; the relative length of the smooth-wall channel of $\xi_{sm} = x/d = 20$; diameter of the channel d =3mm.; the wall temperature $T_W = 25$ °C; the temperature of the fluid at the inlet of a channel $T_0 = 20$ °C.

CONCLUSION

The calculations performed have shown that the greatest values of the effectiveness ratio k_Q, k_N and k_F can be reached at high values of the porosity (θ = 0.8–0.9), at the values of the Reynolds numbers in the channel with smooth wall $Re_{sm} = 10^4 - 2{\cdot}10^4$, at the values of dimensionless length of compared smooth-wall channel approximately $\xi_{sm} = 20$; at a diameter of the channel d < 3 mm and at the magnitude of the difference between the of temperature of the wall of the channel and temperature of the fluid at the inlet in the channel $T_w - T_0 \approx 5$ °C. In this case the quantity of heat, transmitted by the porous channel, in comparison with the quantity of heat, transmitted by the channel with smooth wall is increased by 6 %.

On comparison of the power spent on pumping of the heat transfer medium through the channels, it is possible to achieve the effectiveness ratio by a factors of 1.28. The value of the an increase in dimensional coefficient of effectiveness ratio in this area of parameters reaches the value k_F=3.5.

During calculations a considerable increase in the effectiveness coefficients with the decrease in the diameter of the channel was shown too.

On the basis of the given data it is possible to make an inference, that the application of porous channels with a turbulent motion of a liquid coolant is expedient in removing low of potential heat, with the use of channels of a small diameter and small lengths of compared channels with a smooth wall.

The data obtained points to the necessity of further searching for the parameters, in which there are the energy scoring of porous channels in comparison with traditional channels with a smooth wall.

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