# THE PHENOMENA OF THE HYSTERESIS IN CAPILLARIES

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**Abstract**

Various variants of wetting hysteresis in capillaries are considered. Little changes of geometry or a roughness of the walls of the capillary channel result in a static hysteresis, even in the case of a constant wetting angle. Theoretically it is due to ambiguity of the differential equations solution describing static position of a meniscus and its shape. It was shown that in the case of moving liquid the change in roughness of a surface result in the phenomenon of a serial hysteresis when raising and lowering the liquid level. The effect of low-frequency vibrations on meniscus position in a capillary is considered. The possible mechanism of the anomalously high level of liquid in a capillary during vibrations is suggested.

**KEYWORDS**

Capillary, wetting angle, hysteresis, variable radius, roughness, vibration.

## INTRODUCTION

In a classical design of a heat pipe the liquid motion in a porous structure is provided by capillary pressure, which, in majority of cases, determines the maximum heat power, transmitted by the heat pipe. Hence, the accurate calculation of the capillary pressure is extremely important. In real conditions the geometry of capillary-porous structures is complicated [1], but in simplified approach the porous structure could be considered as a channel with variable cross-section [2].

It is well known the phenomenon of wetting hysteresis [3, 4], which results in different liquid levels in a capillary at the same major parameters. The interest to this phenomenon is also caused by recent research of the vibrations effect on the capillary processes [5–7]. According to [3], hysteresis is caused by kinetic changes of the wetting angles during meniscus motion, and by their relaxation. From multiple hysteresis phenomena we can single out the phenomena which are related only with geometry of the capillary channel. This includes small, in comparison with capillary radius, changes in a cross-section (macroscale) and wall roughness (microscale). Lets consider how it effects the static meniscus position and its motion to equilibrium.

## STATIC HYSTERESIS

The equation, which describes the static meniscus position and its shape in the case of vertical capillary channel with cylindrical symmetry [4]:

$$\frac{h''}{\left(1+h'^2\right)^{3/2}}+\frac{h'}{r\left(1+h'^2\right)^{1/2}}=\frac{\rho g}{\sigma}h, \tag{1}$$

which in the case of constant capillary diameter has a well known approximate solution:

$$h=2\frac{\sigma}{\rho g a}\mathrm{Cos}\,\alpha, \tag{2}$$

here $a$ – capillary radius, m; $g$ – free fall acceleration, m/s$^2$; $h$ – the height of meniscus above the liquid level in container, m; $r$ – current radius, m; $\alpha$ – wetting angle; $\rho$ – liquid density, kg/m$^3$; $\sigma$ – surface tension, N/m; stroke means derivative on $r$.

In the case when radius changes, the equation (1) needs to be solved. In dimensionless form it is

$$\frac{\bar{h}''}{\left(1+\bar{h}'^2 \big/ R_h^2\right)^{3/2}}+\frac{\bar{h}'}{n\left(1+\bar{h}'^2 \big/ R_h^2\right)^{1/2}}=R_h^2\,\frac{\rho g a^2}{\sigma}\,\bar{h}\,. \tag{3}$$

In (3): $\bar{h}=h/a$, $n=r/R(\zeta)$, $R_h=R(\zeta)/a$, $R(\zeta)$ – radius of the capillary cross-section, $\zeta=x/a$ – dimensionless longitudinal coordinate, measured from the liquid level in container.; $a$ is the scale. We will assume that the capillary radius is changing according: $R_h=\left[1+A\mathrm{Sin}\cdot\left(2\pi\Omega\zeta\right)\right]$, where $A$ is the dimensionless amplitude, $\Omega$ is the frequency.

The boundary conditions for the equation (3) equal

$$\bar{h}(0)=h_0\,,\qquad \left.\frac{d\bar{h}}{dn}\right|_0=0\,. \tag{4}$$

Running through $h_0$ one can obtain at $n$=1 values of $\bar{h}(1)$ and derivative $\theta=\bar{h}'(1)$. The calculation is finished when value of $\theta$ coincide with $\mathrm{tg}(\beta)$, where $\beta$ is the angle from the first derivative for meniscus surface. As an example, in Figure 1 the dependence of $\theta$ (curve 2) on tube length, found from solving the equation (3) for the capillary of constant radius ($A$=0), is shown. Number 1 denotes the line $\theta=\mathrm{tg}(\beta)=1/\mathrm{tg}\alpha$. As one can see, there is only one intersection of these lines, which is the solution of this problem.

Lets consider the case $A\neq 0$. In figure 2 values of θ (curve 2) obtained by integration of equation (3) with boundary conditions (4) are shown. Curve 1 corresponds to θ obtained by formula

$$\theta=\mathrm{tg}(\beta)=\frac{1-2\pi A\Omega\,Cos\left(2\pi\Omega\zeta_h\right)\mathrm{tg}\,\alpha}{2\pi A\Omega\,Cos\left(2\pi\Omega\zeta_h\right)+\mathrm{tg}\,\alpha}\,, \tag{5}$$

where $\zeta_h=\bar{h}(1)$, obtained from geometry consideration taking into account the channel radius change. The dependence (5) determines the tangent angle with meniscus surface at the point of contact with channel surface, which is provided by wetting angle. From data shown in Figure 2 it is seen that there are few intersections, i.e. there are several solutions for the problem. Such solutions can be called a static hysteresis. Among these intersections there are stable solutions (curve 2 crosses curve 1 up along axis $\zeta$) and unstable (down). The stable positions are the points of meniscus attraction near these positions. Unstable points divide the attraction domains. The data are shown for the capillary with average radius $a=0.237\cdot 10^{-3}$ м, for the case, when $\mathrm{tg}\alpha=1$, $A=0.04$ and $\Omega$=0.25.

Lets consider another case, when variations of the channel are happening on a microscale in comparison with capillary radius, i.e. variations are caused by changes in surface roughness. Taking into account surface roughness, the effective wetting angle $\alpha_s$ can be calculated as [3]:

$$\mathrm{Cos}\alpha_s=K\cdot\mathrm{Cos}\alpha\,, \tag{6}$$

where $K=S/S_s$ is roughness coefficient, $S$ is area of the smooth surface, $S_s$ is area of the rough surface. Lets consider the case when capillary radius is constant, and roughness coefficient changes according : $K=\left(1+A\mathrm{Sin}\left(2\pi\Omega\xi\right)\right)/(1+A)$. The considered dependence of the roughness coefficient, as in previous case, is required to model heterogeneity of the surface roughness.

In Figure 3 intersections of curves $\theta = \bar{h}^{/}(1)$, which is obtained by solving the equation (3) when varying $h_0$, and $\theta_s$ determined by

$$\theta_s = \frac{K}{\left(1 + tg^2\alpha - K^2\right)^{1/2}} \quad , \tag{7}$$

are shown. $\theta_s$ is calculated on the channel boundary taking into account the effective wetting angle according to (6) (for $K = 1 \quad \theta_s = \mathrm{tg}\beta = 1/\mathrm{tg}\alpha$ , which corresponds to a smooth surface ). Calculations are made for the case $A$=0.04 and $\Omega$=0.25. From data presented in the figure, one can see that there are four stable solutions, i.e. variation of the capillary surface roughness can also lead to ambiguous solutions, i.e. phenomenon of static hysteresis. Thus, in capillaries with variable radius or with variable surface roughness along the channel, there is a possibility of multiple solutions for the static meniscus position, which is hysteresis phenomenon. In larger extent this relates to various cracks, slits or porous media. The fact of such meniscus behavior in capillaries is well known, but it has not been expressed in mathematical terms.

## THE INFLUENCE OF SURFACE ROUGHNESS ON A MOTION OF MENISCUS IN A CAPILLARY

One of the classical problems related to capillary effect is the one considering a rise of the liquid in capillary. Substantial interest to this problem stems from the presence of wetting hysteresis. In accordance with [3] , there are several forms of hysteresises, that are connected in one or other way to the difference between the observed wetting angle and its equilibrium value calculated using Young equation. Contact angles are formed under the influence of different inter-phase and dynamic forces. There are two types of contact angles in the presence of friction: advancing angles and receding angles, and the difference between the two angles can be as high as several dozen degrees. It is possible to eliminate the hysteresis effect in a case of clean and extremely smooth surfaces. In a case where the surface is rough, the hysteresis effect immediately manifests itself.

In the above, the hysteresis effect has been presented as multiple solutions for correspondent equations defining static position of the meniscus. Now, we can demonstrate how the roughness of capillary walls effects the static position of the wetting moving liquid. Due to the fact that we take into account only the influence of pure geometrical properties of the capillary channel, we can consider that wetting angle has constant value and is not related to the parameters describing the motion of liquid.

Mathematical problem for the meniscus motion was thoroughly formulated in [8]. In this paper, the equations describing the motion of columns of liquid in a capillary have been derived from general laws of mechanics. It is worth to mention a recent publication [9], where authors modeled the process where one liquid column pushes another in a capillary. In this publication, the mathematical problem for meniscus motion is solved in some simplified way. In this paper we will use our own derivation of motion equation, this derivation is more convenient for solution of subsequent problems. To derive the equation for liquid motion in a capillary, we will use approximation, described in [10] as well as equation for boundary layer, which well describes motion of gases or liquids in thin layers. As a result of not very difficult manipulations, one can obtain a following simple equation

$$\frac{dp}{dx} = -\rho\frac{du}{dt} - 8\frac{\mu}{a^2}u - \rho g\,, \tag{8}$$

where $t$ is time, $p$ is pressure, $u$ is the velocity averaged over the channel cross section, and $\mu$ is the dynamic viscosity.

After integration of the equation (8) of $x$ from lower edge of capillary to the upper edge of liquid and by neglecting the changes of meniscus height compared to the total height $h$ of the layer, one can obtain

$$p_h - p_0 = -\rho h \frac{du}{dt} - 8\frac{\mu}{a^2} hu - \rho g h\,, \tag{9}$$

where $p_0$ is the pressure at lower edge of capillary, $p_h$ is the pressure averaged over radius. From the Laplace equation averaged over cross section, it follows that

$$p_h - p_a = \frac{2}{a^2}\sigma \int_0^a r\left( \frac{h_{rr}^{//}}{\left(1+h_r^{/}\right)^{3/2}} + \frac{h_r^{/}}{r\left(1+h_r^{/}\right)^{1/2}} \right) dr = 2\frac{\sigma}{a}\frac{\theta}{\left(1+\theta^2\right)^{1/2}}\ , \tag{10}$$

where $p_a$ is atmospheric pressure. Taking into account that $u=dh/dt$, finally we can obtain following expression

$$h\frac{d^2h}{dt^2} + gh + 8\frac{\mu}{\rho a^2} h\frac{dh}{dt} - 2\frac{\sigma}{a}\frac{\theta}{\left(1+\theta^2\right)^{1/2}} = p_0 - p_a\ . \tag{11}$$

Figure 4 demonstrates the curves describing raise and drop of liquid meniscus for smooth channel wall for condition where tgα = 1. It can be seen from this Fig. 4 that the curves behave differently depending on the initial height of the liquid layer, however, the static height is the same for all the curves and corresponds to the height calculated from Eq (2).

Let us assume that the roughness is changing in accordance with the law described in the above. Figure 5 shows the result of calculations for $A$ = 2 and Ω = 1. In the case of capillary radius of $0.225\cdot10^{-3}$ m, one can observe a hysteresis phenomenon, that is when initial level of meniscus is different, two different heights can be achieved, which are then constant. When liquid is moving upward in the capillary, the meniscus stops at one level, and when the liquids moves downward, the meniscus stops at some higher level.

Therefore, taking into account only wall roughness on the effectual wetting angle and neglecting other factors influencing the wetting, we can observe the hysteresis for the liquid moving in the capillary.

## THE VIBRATIONS EFFECT ON A MENISCUS MOVEMENT IN A CAPILLARY

The anomalous increase of liquid height in a capillary during low frequency vibrations is described in [5–7]. Let’s consider liquid motion in a vibrating capillary using previously described approach.

In this case the equation (11) in coordinate system related to capillary will take a form

$$h\left(\frac{d^2h}{dt^2} + \frac{dU_c}{dt}\right) + gh + 8\frac{\mu}{\rho R_c^2} h\frac{dh}{dt} - 2\frac{\sigma}{R_c}\frac{\theta}{\left(1+\theta^2\right)^{1/2}} = p_0 - p_a\,, \tag{12}$$

where $U_c$ is the velocity of motion of the capillary pipe. In equation (12) the pressure at the bottom section of the pipe is unknown. To find it, the problem of liquid motion in a container with placed in it liquid column was solved. As a result of approximate solution of this problem from Lagrange integral [11] in a linear approximation using Bessel functions, the analytical dependence of the pressure in liquid for harmonic motion was obtained.

The results of numeric solution of the equation (12) are shown in Figure 6. The amplitude of vibrations was $1\cdot10^{-3}$ m, frequency – 40 Hz. The rest of the parameters were the same, as in described above example. Numbers 1, 2 denotes curves corresponding lift and descent of liquid meniscus without vibrations. Curves 3, 4 corresponds to the case with vibrations. It can be seen (Fig. 6 a) that during lift and descent the liquid stops at different positions. When vibrations are present the meniscus positions become closer. It is well shown in Fig. 6 b, where straight lines corresponds to meniscus positions without vibrations (curves 1, 2). After 60 s vibrations were turned off and meniscus took a static position inside the domain limited by curves 1, 2. The difference in their position is greatly reduced in this case. Hence, vibrations reduce the effect of surface roughness and practically remove hysteresis phenomenon. This classic result is well known in the literature [4].

It was shown [6, 7] that at relatively low frequencies of 40-80 Hz the anomalous rise of liquid is taking place. There is no currently a reasonable explanation of such effect. Using the proposed approach to hysteresis phenomenon, we can suggest possible explanation. It is known, that during meniscus motion the dynamic wetting angle changes. If the assume, that vibrations change the wetting angle, the value of $\theta$ in equations (11), (12) will also change. We can write this dependence in the form of $\theta = \theta_{st} + \chi u^2$, where $\theta_{st}$ corresponds to static position, which was used above, $\chi$ is some coefficient. The calculation results are shown in Figure 7. Curves 1, 2, as before, corresponds to the case without vibrations. Curves 3, 4 shows the meniscus dynamics with θ dependence ($\chi$ was equal 100). The meniscus takes in this case higher position, than the position of the top static state (curve 2). When vibrations are turned off, meniscus goes along curve 2, i. e. take the static position defined by serial hysteresis. Thus, the mechanism of anomalous rise of the liquid during vibrations could be related to dynamic effects, which affect the wetting angle.

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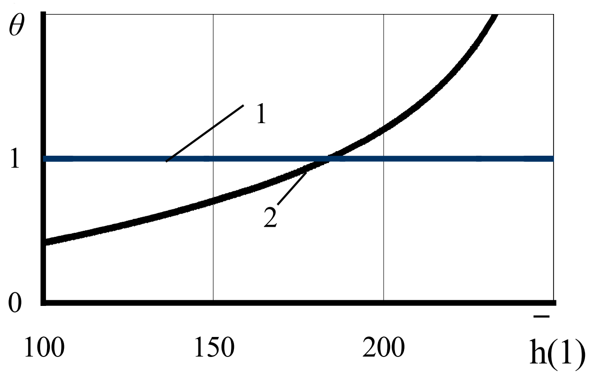


Fig. 1. Curves θ at $A = 0$. 1 – $\theta = tg\beta$, 2 – from the decision of the equation (3)

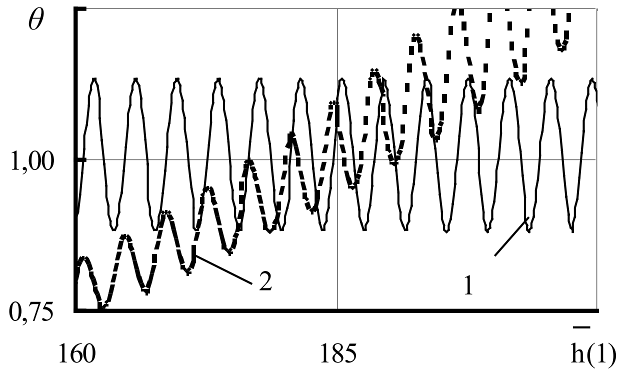


Fig. 2. Curves θ at $A \neq 0$. 1 – under the formula (5), 2 – from the decision of the equation (3)

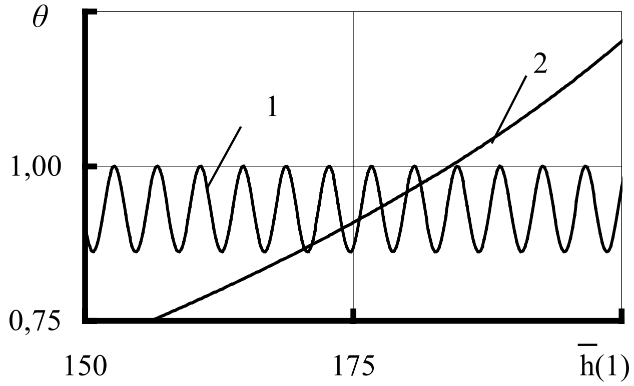


Fig. 3. Curves θ for a rough surface. 1 – under the formula (7), 2 – from the decision of the equation (3)

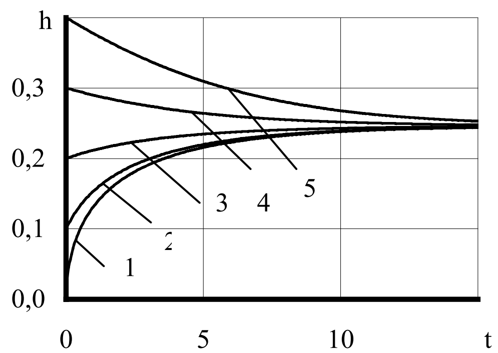


Fig. 4. Movement of a meniscus of a liquid in a capillary which has smooth walls.
$A = 0.225 \cdot 10^{-3}$ m, $tg\alpha = 1$.
1 – $h_o = 0$ m, 2 – $h_o = 0.1$ m, 3 – $h_o = 0.2$ m, 4 – $h_o = 0.3$ m, 5 – $h_o = 0.4$ m

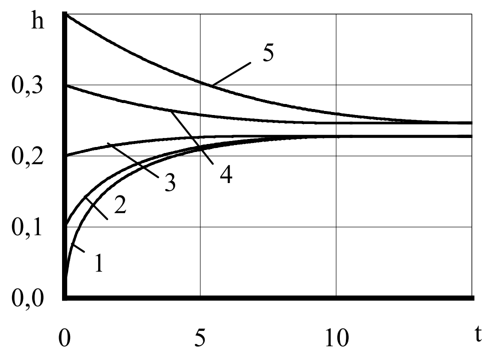


Fig. 5. Movement of a meniscus of a liquid in a capillary which has rough walls.
$A = 0.225 \cdot 10^{-3}$ m, $tg\alpha = 1$.
1 – $h_o = 0$ m, 2 – $h_o = 0.1$ m, 3 – $h_o = 0.2$ m, 4 – $h_o = 0.3$ m, 5 – $h_o = 0.4$ m

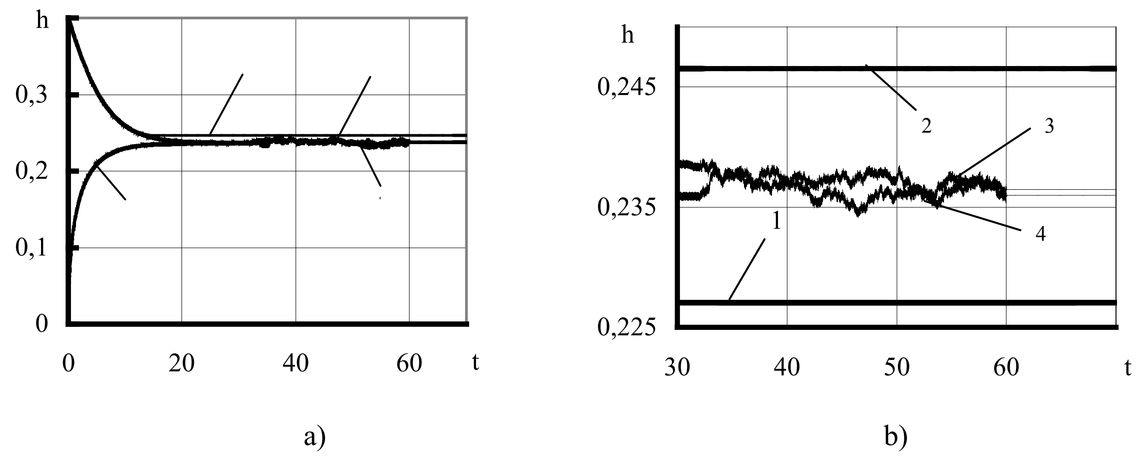


Fig. 6. Effect of low-frequency vibrations on meniscus position in a capillary. 1, 2 – without vibration; 3, 4 – with vibration, $A = 1 \cdot 10^{-3}$ m, $f = 40$ Hz; 1, 3 – rise ($h_o$=0 m); 2, 4 – lowering ($h_o$ = 0.4 m)

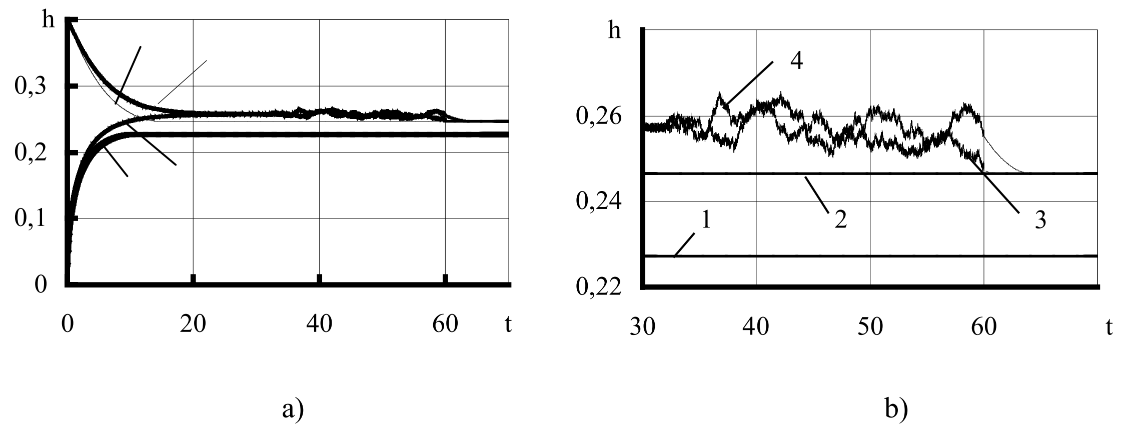


Fig. 7. Effect of low-frequency vibrations on meniscus position in a capillary with dynamic change of a wetting angle. 1, 2 – without vibration; 3, 4 – with vibration, $A$=$1 \cdot 10^{-3}$ m, $f$=40 Hz; 1, 3 – rise ($h_o$=0); 2, 4 – lowering ($h_o$=0,4 m)