CONNECTIVE HEAT TRANSFER IN MICROTUBES FOR SLIP FLOW

Sadik Kakaç[1], Almila Yazicioglu[2], Wei Sun[3]

[1]TOBB University of Economics and Technology, Ankara, Turkey
[2]Middle East Technical University, Ankara, Turkey
[3]University of Miami, Coral Gables, FL, USA

Abstract

The steady-state convective heat transfer in laminar, hydrodynamically developed flow in microtubes with constant wall temperature and constant heat flux boundary conditions are solved by the numerical method. Temperature jump and velocity slip conditions and viscous heating are included. The solution is verified for the cases where viscous heating is neglected. For these two cases, with a given Brinkman number, at specified axial lengths, viscous effects are presented for the thermal entrance length. Also, viscous heating is investigated for all the cases where the fluid is being heated or cooled. The effects of the Knudsen, Brinkman and Prandtl numbers are presented in the fully developed conditions and in the thermal entrance region in graphical and tabular forms. The axial conduction effect is also indicated.

KEYWORDS

Microtubes, Micro Heat Transfer, Viscous Heating, Axial Conduction

INTRODUCTION

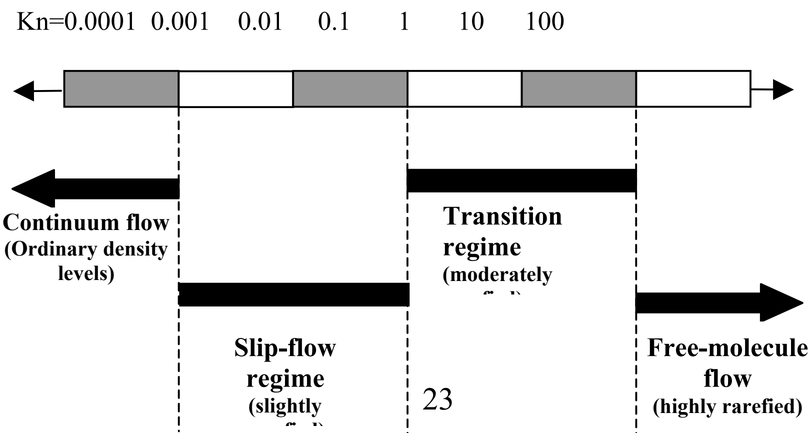
With the increase of integrated circuit density and power dissipation of microelectronic devices, it is becoming more necessary to employ effective cooling devices and cooling methods to maintain the operating temperature of electronic components at a safe level. New thermal control methods have become mandatory as existing heat flux levels exceed 100 W/cm^2 [1, 2].

There are basically two ways of modeling a flow field in microchannels, which depend on the Knudsen number. Knudsen number is proportional to (length of mean free path) / (characteristic dimension) and is used in momentum and mass transfer in general and very low pressure gas flow calculations in particular. It is normally defined in the following form [3]:

$$\mathrm{Kn} = \frac{\lambda}{D}, \tag{1}$$

where λ is the length of mean free path of the molecules. D is the characteristic dimension. Generally the traditional continuum approach is valid, albeit with modified boundary conditions as long as Kn<0.1.

From Fig. 1, we know that for the small values (Kn≤10^{-3}), the fluid is considered to be continuum, while for large values (Kn≥10), the fluid is considered to be a free molecular flow. For 10^{-3}< Kn<10^{-1} is the near continuum region.



Another dimensionless number is the Brinkman number, which is the ratio of heat generated by viscous dissipation to heat transferred by conduction over the cross-section, which is used in the energy equation considering temperature jump on the wall in general and very low pressure gases in particular. It is normally defined in the following form:

$$\mathrm{Br} = \frac{\mu u_m^2}{k\Delta T}, \tag{2}$$

where μ is dynamic viscosity of flow in microtubes, u_m is sectionally-averaged velocity of the flow in microtubes, k is the axially local and sectionally-averaged thermal conductivity coefficient, and ΔT is axially local wall-flow temperature difference (T_0-T_s).

Gaseous flow in microchannels was experimentally analyzed by Shih et al. (1996) [5] with helium and nitrogen as the working fluids. Mass flow rate and pressure distribution along the channels were measured. Helium results agreed well with the result of a theoretical analysis using slip flow conditions, however there were deviations between theoretical and experimental results for nitrogen.

Tunc and Bayazitoglu (2000 and 2001) [6, 7] studied the convective heat transfer for steady state, laminar, hydrodynamically developed flow in microtubes with uniform temperature and uniform heat flux boundary conditions by the use of the integral transform technique. In another paper Tunc and Bayazitoglu (2001) [8] considered heat transfer by convection in a rectangular microchannel. The flow is assumed to be fully developed both thermally and hydrodynamically. The H2-type boundary condition, constant axial and peripheral heat flux, is applied at the walls of the channel.

ANALYSIS

Constant wall temperature

In micro-channels, collisions of gas molecules with the flow boundary will occur more often than those between molecules. Moreover, macro-channel boundary conditions are different from the typical ones (no temperature difference between the wall and the flow and the relative velocity along the wall is zero). There exist a slip velocity along the wall and temperature jump between wall and fluid.

We analyze two-dimensional heat transfer in micro tubes with constant fluid properties. Since the fluid next to the wall has a temperature finitely different from the wall temperature, slip temperature should be used for the first boundary condition. The second boundary condition is the centerline symmetry and uniform temperature at the channel entrance.

$$\text{Slip velocity:} \quad u_s = -\frac{2-F_m}{F_m}\lambda\left(\frac{du}{dr}\right)_{r=R}. \tag{3}$$

In this relation, u_s is the slip velocity, λ the molecular mean free path, and F_m is the momentum accommodation coefficient. It is a function of the interaction between gas molecules and the surface. If the surface is absolutely smooth and reflect the molecules specularly, the tangential momentum will not change. Therefore F_m will be equal to zero. Molecules can also be reflected diffusely, which results in $F_m = 1$. This means that all the tangential momentum is lost at the wall [6].

The fully developed velocity profile for slip flow is derived from the momentum equation using the slip velocity as

$$u = \frac{2u_m\left[1-\left(\frac{r}{R}\right)^2 + 4\mathrm{Kn}\right]}{1+8\mathrm{Kn}}, \tag{4}$$

where u_m is the mean velocity.

$$\text{Temperature jump:} \quad T_s - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{(\gamma+1)}\frac{\lambda}{\mathrm{Pr}}\frac{\partial T}{\partial r}, \tag{5}$$

where T_s is the temperature of the flow at the wall, T_w is the wall temperature, and F_t is the thermal accommodation coefficient is defined as $F_t = \dfrac{(E_a - E_l)}{(E_a - E_w)}$, where E_a is the energy of the approaching stream, E_l is the energy carried by the molecules leaving the surface and E_w is the energy of the molecules leaving the surface at the wall temperature. F_t can be defined as the fraction of molecules reflected by the wall that accommodated their energy to the wall temperature if we have a rough surface or if we have a phase change.

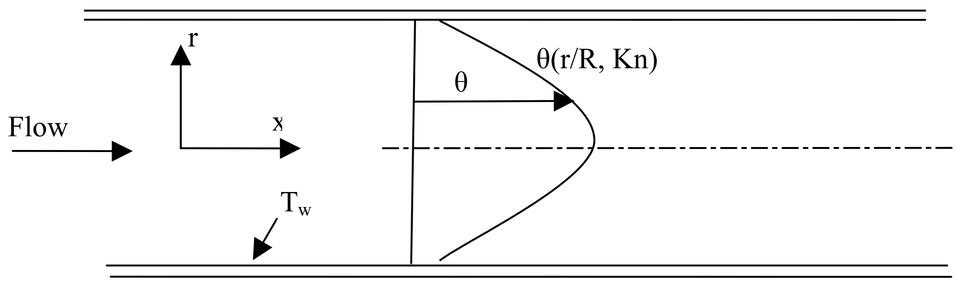


Fig. 2. Sketch of the geometry

Energy equation under fully developed velocity, with constant fluid properties including heat dissipation can be written as

$$u\frac{\partial T}{\partial x}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\nu}{c_p}\left(\frac{du}{dr}\right)^2, \tag{6}$$

$$\text{at r = R} \quad T=T_s, \tag{6 a}$$

$$\text{at r = 0} \quad \frac{\partial T}{\partial r}=0, \tag{6 b}$$

$$\text{at x = 0} \quad T=T_0. \tag{6 c}$$

The energy equation (6) can be made dimensionless by defining the following dimensionless numbers:

$$\theta=\frac{T-T_s}{T_0-T_s},\ \eta=\frac{r}{R},\ \varsigma=\frac{x}{L},\ u^*=\frac{u}{u_m}=\frac{2\left(1-\eta^2+4\text{Kn}\right)}{1+8\text{Kn}}. \tag{7}$$

Then, we obtain the following dimensionless form of the energy equation (6):

$$\frac{Gz\left(1-\eta^2+4\text{Kn}\right)}{2\left(1+8\text{Kn}\right)}\frac{\partial\theta}{\partial\zeta}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right)+\frac{16\text{Br}}{\left(1+8\text{Kn}\right)^2}\eta^2, \tag{8}$$

$$\eta=1 \qquad \theta=0, \tag{8 a}$$

$$\eta=0 \qquad \frac{\partial\theta}{\partial\eta}=0, \tag{8 b}$$

$$\zeta=0 \qquad \theta=1, \tag{8 c}$$

where Gz is the Graetz number defined as $Gz=\frac{\text{Re}\,\text{Pr}\,D}{L}$, and Br is the Brinkman number defined as $\text{Br}=\frac{\mu u_m^2}{k\Delta T}$, where $\Delta T=T_0-T_s$ is the temperature difference between the fluid inlet and tube wall.

Heat transfer from the fluid to the wall by convection is written as

$$q_x=h_x\left(T_b-T_w\right). \tag{9}$$

Heat flux at the wall can also be written using Fourier's law

$$q_x=-k\left.\frac{\partial T}{\partial r}\right|_{r=R}, \tag{10}$$

where the bulk temperature is defined as

$$\theta_b=2\int_0^1\left(\frac{u}{u_m}\right)\theta\left(\eta,\zeta\right)\eta d\eta. \tag{11}$$

The Nusselt number is obtained as follows [12]:

$$Nu_x = \frac{h_x D}{k} = -\frac{2\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}}{\left(\theta_b - \dfrac{4\gamma}{\gamma+1}\dfrac{Kn}{\Pr}\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}\right)} . \qquad (12)$$

Constant heat flux

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\nu}{c_p}\left(\frac{du}{dr}\right)^2 , \qquad (13)$$

$$\text{at r = R} \qquad k\frac{\partial T}{\partial r} = q'' , \qquad (13\text{ a})$$

$$\text{at r = 0} \qquad \frac{\partial T}{\partial r} = 0 , \qquad (13\text{ b})$$

$$\text{at x = 0} \qquad T = T_0 . \qquad (13\text{ c})$$

The energy equation (13) can be made dimensionless by defining $\theta = \dfrac{T - T_0}{q''R/k}$

$$\frac{Gz\left(1-\eta^2+4Kn\right)}{2\left(1+8Kn\right)}\frac{\partial\theta}{\partial\zeta} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right) + \frac{32Br}{\left(1+8Kn\right)^2}\eta^2 , \qquad (14)$$

$$\eta = 1 \qquad \frac{\partial\theta}{\partial\eta} = 1 , \qquad (14\text{ a})$$

$$\eta = 0 \qquad \frac{\partial\theta}{\partial\eta} = 0 , \qquad (14\text{ b})$$

$$\zeta = 0 \qquad \theta = 0 , \qquad (14\text{ c})$$

where Br is defined as $Br = \dfrac{\mu u_m^2}{q''D}$.

Similarly, we can derive an expression for the local Nusselt number as above [12]

$$Nu_x = \frac{h_x D}{k} = \frac{2}{\theta_s + \dfrac{4\gamma}{\gamma+1}\dfrac{Kn}{\Pr} - \theta_b} . \qquad (15)$$

NUMERICAL METHOD

With the development of high-speed digital computers, numerical techniques have been developed and extended to handle almost any problem of any degree of complexity. The finite volume method [10, 11] is employed to solve the energy equation under the boundary conditions mentioned.

RESULTS AND DISCUSSIONS

The energy equation including viscous dissipation has been solved numerically under different boundary conditions. The fully developed and the local Nusselt numbers are determined and the effects of the Knudsen number, the Brinkman number and the Prandtl number on heat transfer are presented and details are given in [13–15]. The analytical and numerical results are compared in Table 1 with the data available in the literature to show the validity of the present numerical solutions.

Table 1. Developed conditions, laminar flow nusselt values (T_w = constant, Pr = 0.6)

Br = 0	Nu_T (analytical)	Nu_T (numerical)
Kn = 0.0	3.6751	3.6566
Kn = 0.02	3.3675	3.3527
Kn = 0.04	3.0745	3.0627
Kn = 0.06	2.8101	2.8006
Kn =0.08	2.5767	2.5689
Kn =0.10	2.3723	2.3659

Kn =0.12	2.1937	2.1882

The results are shown in Table 2 and Fig. 3 for uniform wall temperature boundary conditions with Br = 0.01. The fully developed Nusselt number decreases as Kn increases. For the no-slip condition Nu = 9.5985, while it drops down to 3.4016 for Kn = 0.12 under Pr = 0.7, a decrease of 64.5 %. This is due to the fact that the temperature jump reduces heat transfer. As Kn increases, the temperature jump also increases. Moreover, in Fig. 3 the fully developed Nusselt number increases as Pr increases, due to the same reason; temperature jump, which decreases as Pr increases, reduces heat transfer. Therefore, the denominator of eq.(14) takes smaller values.

Table 2. The fully developed Nusselt number values (Pr = 0.7, T_w = constant)

Pr = 0.7	Br = 0.00	Br = 0.01
Kn = 0.00	3.6566	9.5985
Kn = 0.02	3.4163	7.4270
Kn = 0.04	3.1706	6.0313
Kn = 0.06	2.9377	5.0651
Kn = 0.08	2.7244	4.3594
Kn = 0.10	2.5323	3.8227
Kn = 0.12	2.3604	3.4016

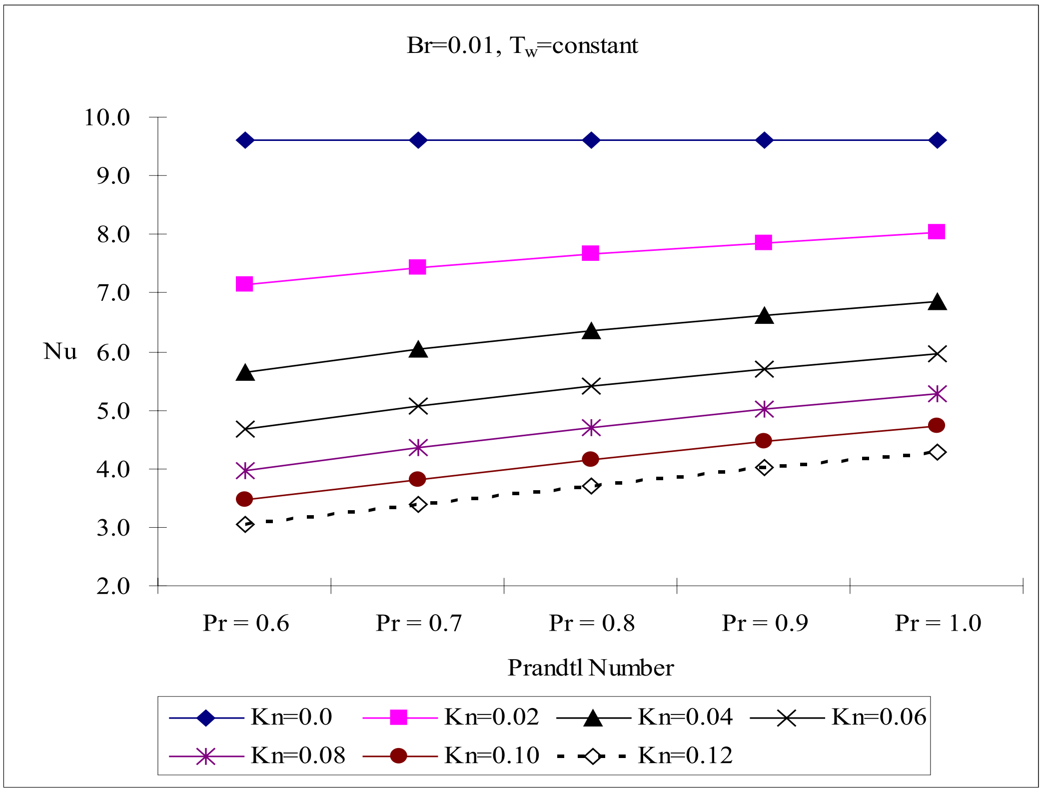


Fig. 3. The effect of Prandtl number on heat transfer for constant T_w

The variation of the fully developed Nusselt number with Kn for two cases, with and without viscous heating, is given Fig. 4. It is seen from the figure that, for the uniform temperature boundary condition at the wall, Kn number increase has more influence with the presence of viscous dissipation. Also, the Nusselt number has a larger value for the case with viscous heating.

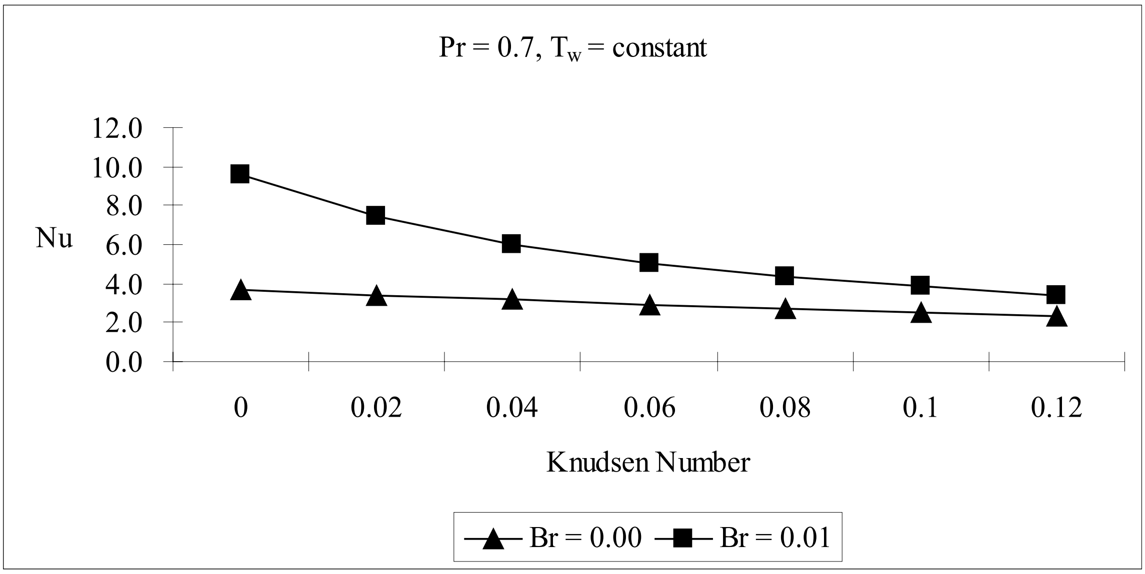


Fig. 4. The effect of Brinkman number on heat transfer for constant T_w

The results are shown in Table 3 for uniform heat flux boundary conditions with Br = -0.01. The fully developed Nusselt number decreases as Kn increases. For the no-slip condition Nu = 4.5640, while it drops down to 2.7468 for Kn = 0.12, Br=- 0.01 under Pr = 0.7, a decrease of 39.8%. This is due to the fact that the temperature jump reduces heat transfer. As Kn increases, the temperature jump also increases.

Table 3. The fully developed Nusselt number values, Br = - 0.01, q_w = constant

Br = - 0.01	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	4.5640	4.5640	4.5640	4.5640	4.5640
Kn = 0.02	4.1278	4.2212	4.2953	4.3565	4.409
Kn = 0.04	3.7154	3.8695	3.9948	4.0995	4.1894
Kn = 0.06	3.3484	3.5395	3.6985	3.8338	3.9511
Kn = 0.08	3.0302	3.2419	3.4218	3.5772	3.7137
Kn = 0.10	2.7567	2.9784	3.1701	3.3381	3.4871
Kn = 0.12	2.5219	2.7468	2.9441	3.1191	3.2759

Table 4, shows the effect of positive or negative Br values ($Br = \pm 0.01$) on heat transfer. From the definition of Br, for this type of boundary condition, a negative Br means that the fluid is being cooled. Therefore, the Nusselt number takes higher values for Br < 0 and lower values for Br > 0.

Table 4. The fully developed Nusselt number values, Pr =0.7, q_w = constant

Pr = 0.7	Br = 0.00	Br = 0.01	Br = - 0.01
Kn = 0.00	4.3649	4.1825	4.5640
Kn = 0.02	4.1088	4.0022	4.2212
Kn = 0.04	3.8036	3.7398	3.8695
Kn = 0.06	3.4992	3.4598	3.5395
Kn = 0.08	3.2163	3.1912	3.2419
Kn = 0.10	2.9616	2.945	2.9784
Kn = 0.12	2.7354	2.7242	2.7468

In Fig. 5, we show the effect of temperature jump on the Nusselt number clearly. The solid and dashed lines represent the results from the present study for constant heat flux and constant temperature boundary conditions, respectively. When the temperature jump condition is not considered, in other words, only the velocity slip condition is taken into account, the Nusselt number increases with increasing Kn, which implies that the velocity slip and temperature jump have opposite effects on the Nusselt number.

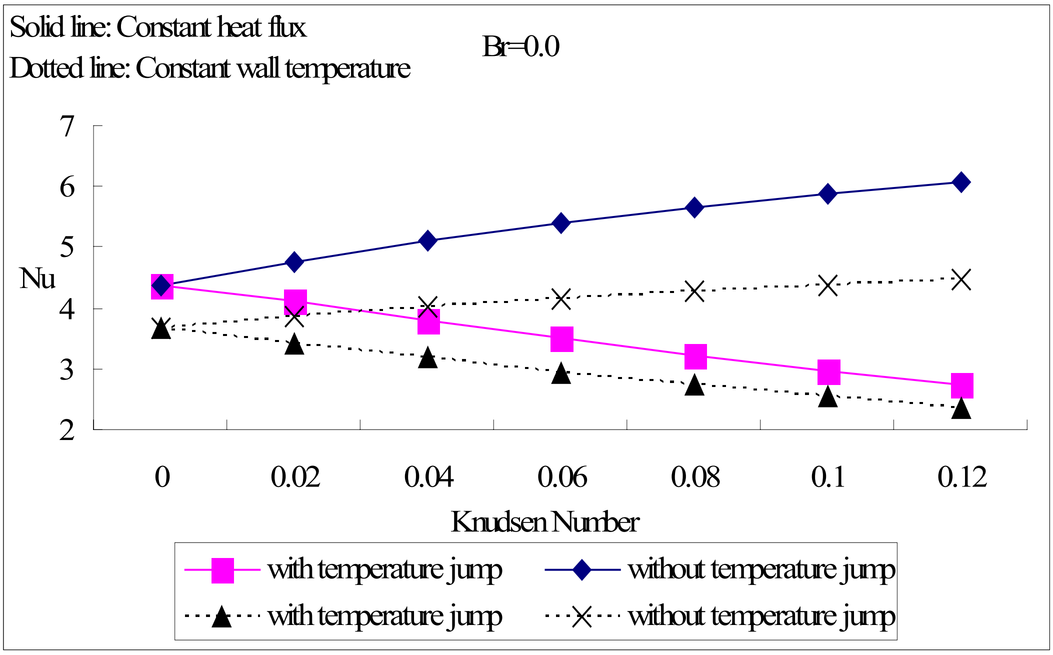


Fig. 5. The effect of temperature jump on heat transfer

In Table 4 and Fig.6, we show the Nusselt number values in the thermally developing rangion. For both of the cases, as Kn increases, the Nusselt number decreases due to the increasing temperature jump. We note here that, the decrease is greater and reaching developed region is later when we consider viscous dissipation. The Nusselt number reaches the developed value as if there is no viscous heating.

Table 4. Variation of the Nusselt number with Kn for uniform T_w

$\frac{x/D}{\mathrm{Re\,Pr}}$		0.001	0.006	0.01	0.06	0.1	0.6	1
Br = 0.0	Kn = 0.0	6.1614	3.9092	3.7152	3.6566	3.6566	3.6566	3.6566
	Kn = 0.04	4.7628	3.3319	3.2052	3.1706	3.1706	3.1706	3.1706
	Kn = 0.08	3.7591	2.8308	2.7461	2.7244	2.7244	2.7244	2.7244
	Kn = 0.12	3.0676	2.4339	2.3749	2.3604	2.3604	2.3604	2.3604
Br = 0.01	Kn = 0.0	6.1658	3.9181	3.7275	4.0997	7.2072	9.5985	9.5985
	Kn = 0.04	4.764	3.3352	3.21	3.4154	5.1649	6.0313	6.0313
	Kn = 0.08	3.7595	2.8322	2.7482	2.8656	3.9352	4.3594	4.3594
	Kn = 0.12	3.0677	2.4346	2.3759	2.4464	3.1516	3.4016	3.4016

In Table 5, the Nusselt number values in the thermally developing range are shown. For all the cases, as Kn increases, the Nusselt number decreases due to the increasing temperature jump.

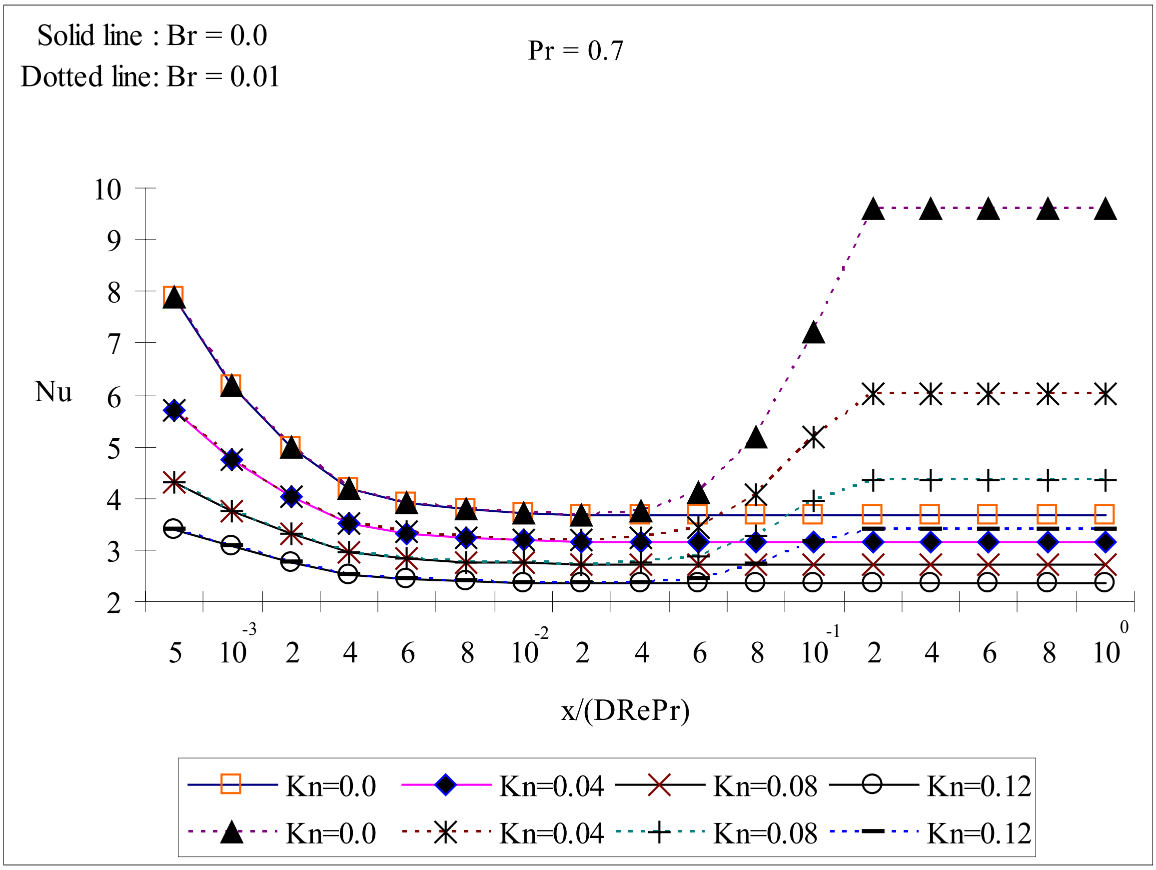


Fig. 6. Variation of the Nusselt number with the Knudsen number at the entrance region for uniform temperature at the wall with temperature jump

Table 5. Nusselt number with the effect of viscous heating at the entrance for uniform q_w

$\frac{x/D}{\mathrm{Re\,Pr}}$		0.001	0.006	0.01	0.06	0.1	0.6	1
Br = 0.0	Kn = 0.0	7.5884	4.8272	4.5196	4.3649	4.3649	4.3649	4.3649
	Kn = 0.04	5.8036	4.0975	3.8872	3.8036	3.8036	3.8036	3.8036
Br = 0.01	Kn = 0.0	7.3531	4.6346	4.3336	4.1825	4.1825	4.1825	4.1825
	Kn = 0.04	5.7474	4.0335	3.8231	3.7398	3.7398	3.7398	3.7398
Br =-0.01	Kn = 0.0	7.8392	5.0365	4.7223	4.564	4.564	4.564	4.564
	Kn = 0.04	5.8608	4.1635	3.9535	3.8695	3.8695	3.8695	3.8695

In Fig.7, the system first reaches the fully developed condition as if there is no viscous heating. Then, at some point, Nu makes a jump to its final value. As Br increases, the jump occurs at a shorter distance from the entrance. Since the wall temperature is constant, they all converge to the same fully developed value. This effect can be explained by the same reason we mentioned before.

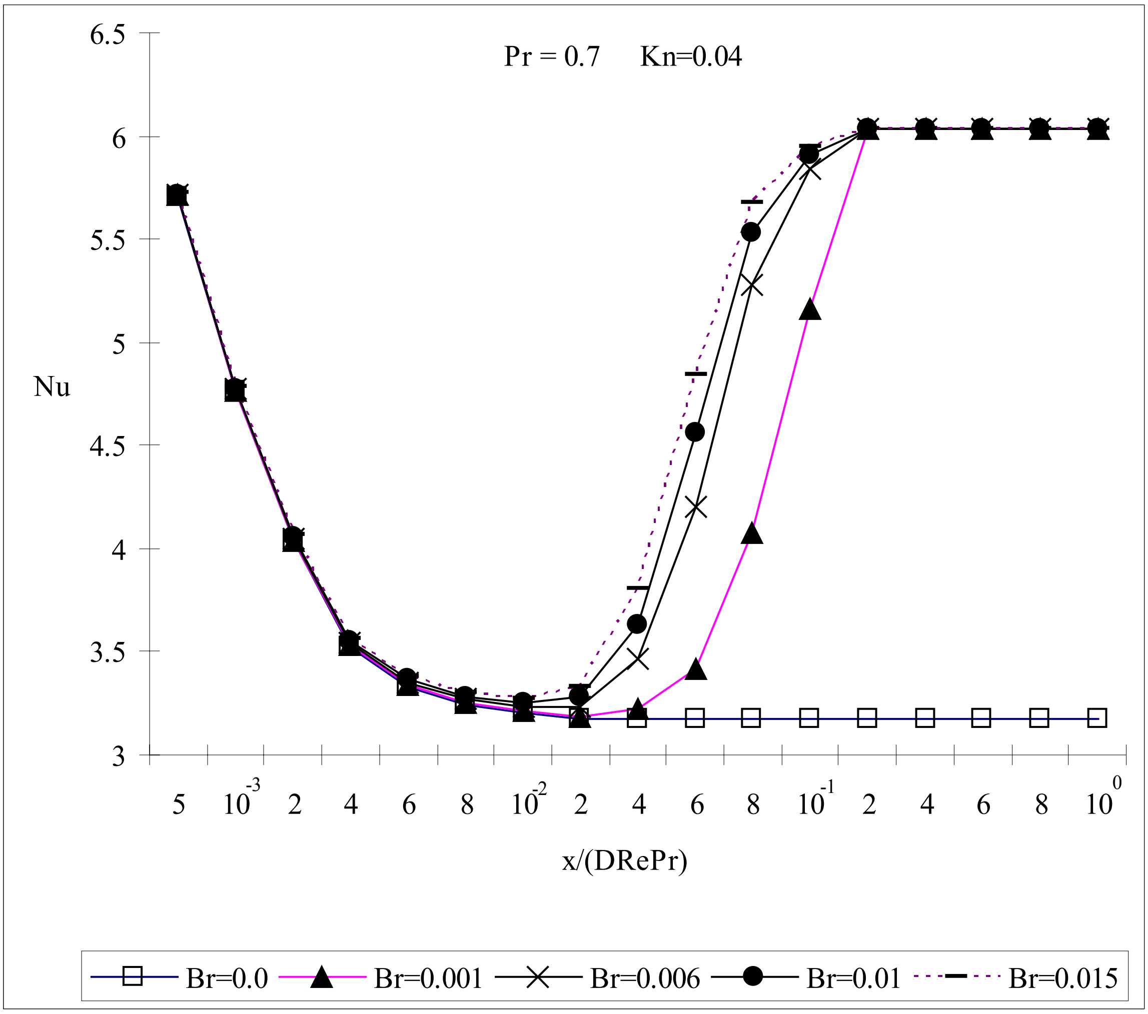


Fig. 7. Variation of the Nusselt number with the Knudsen number at the entrance region for uniform T_w with temperature jump

Table 6 and Fig. 8 indicate the values of the local Nusselt number for constant heat flux boundary condition for various values of the Br number at Kn = 0.04. Since the definition of the Brinkman number is different for the uniform heat flux boundary condition case, a positive Br means that the heat is transferred to the fluid from the wall as opposed to the uniform temperature case. Therefore, we see in Fig. 8 that Nu decreases as Br increases when Br > 0, Negative Br means the flow is cooled and Nu has the larger value.

Table 6. Nusselt number with the effect of viscous heating, uniform q_w

$\frac{x/D}{Re\,Pr}$	0.001	0.006	0.01	0.06	0.1	0.6	1
Br = - 0.01	5.8608	4.1635	3.9535	3.8695	3.8695	3.8695	3.8695
Br = 0.0	5.8036	4.0975	3.8872	3.8036	3.8036	3.8036	3.8036
Br = 0.001	5.7979	4.091	3.8807	3.7971	3.7971	3.7971	3.7971
Br = 0.006	5.7698	4.0589	3.8485	3.765	3.765	3.765	3.765
Br = 0.01	5.7474	4.0335	3.8231	3.7398	3.7398	3.7398	3.7398

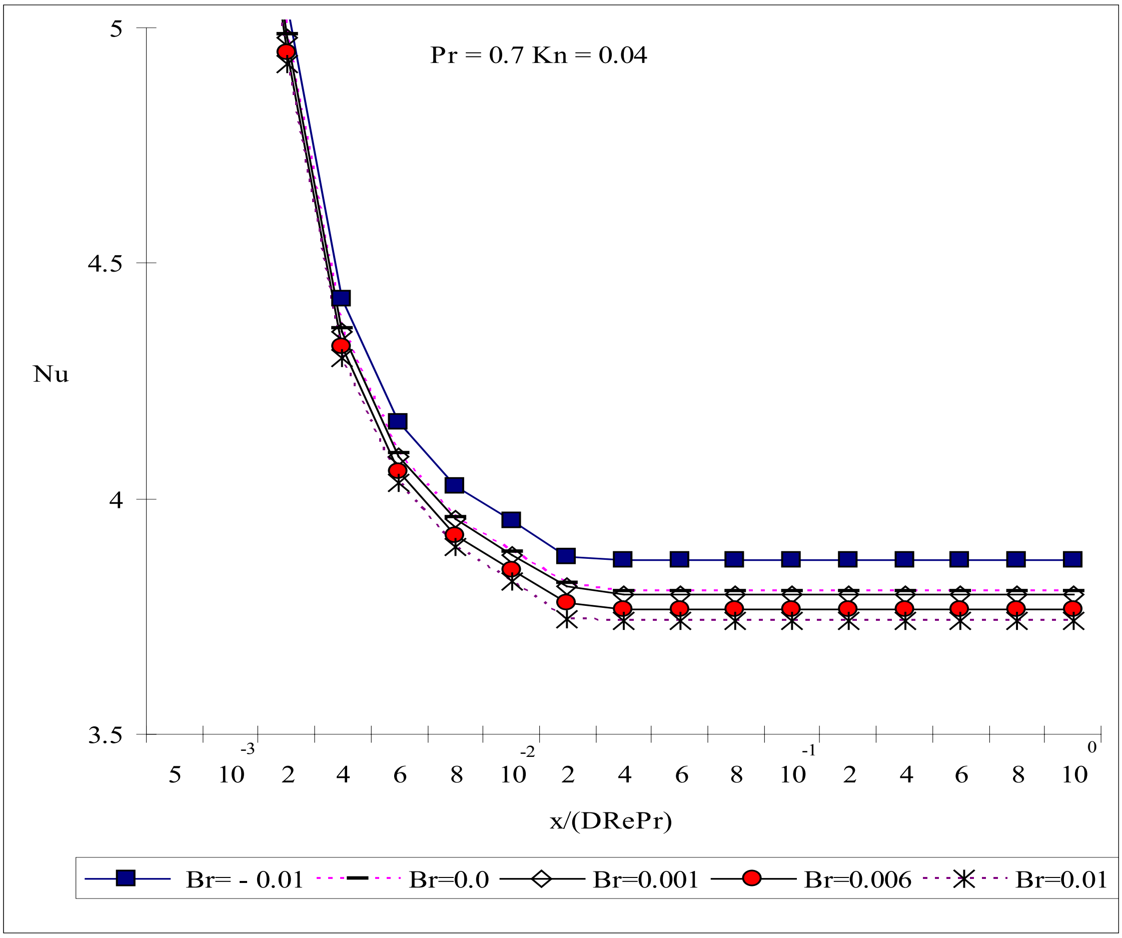


Fig. 8. Variation of the Nusselt number with the Knudsen number at the entrance region for uniform heat flux at the wall with temperature jump

CONCLUSIONS

The effects of three variables, Kn, Pr and Br numbers on flow are discussed. Kn is ranging from Kn = 0, which is the continuum case (macrochannel flow) to Kn = 0.12 which is the upper limit of the slip flow regime, and Prandtl number varied from Pr =0.6 to Pr = 1.0 which are typically for air. Brinkman number is varied from Br = - 0.01 to Br = 0.015.

For uniform wall temperature boundary condition with temperature jump, the fully developed Nu decreases as Kn increases, and increases as Pr increases. Kn increase is due to the reduction of the channel size and has more influence with the presence of viscous dissipation than Br = 0. On the other hand, without temperature jump, the fully developed Nu increases as Kn increases, and keeps unchanged as Pr increases. Still, Kn has more influence with the presence of viscous dissipation.

For uniform heat flux boundary condition with temperature jump, the fully developed Nu decreases as Kn increases and increases as Pr increases. Since the definition of the Br is different for the uniform heat flux boundary condition case; a positive Br means that the heat is transferred to the fluid from the wall as opposed to the uniform temperature case. Nu takes higher values for Br < 0 and lower values for Br > 0. On the contrary, without temperature jump, the fully developed Nu increases as Kn increases and does not change as Pr increases. The Nu takes higher values for Br < 0 and lower values for Br > 0. From the developing region to developed region, the system drops down to reach the fully developed condition. Nu decreases as Br increases when Br > 0.

In general, for the above two boundary conditions, velocity slip and temperature jump affect the heat transfer in opposite ways: a large slip on the wall will increase the convection along the surface. On the other hand, a large temperature jump will decrease the heat transfer by reducing the temperature gradient at the wall. Therefore, neglecting temperature jump will result in the overestimation of the heat transfer coefficient.

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Nomenclature

Br – Brinkman number, Ec/Pr, $\mu u_m^2/k\,\Delta T$
c_p – specific heat at constant pressure, J/(kg·K)
c_v – specific heat at constant volume, J/(kg·K)
D – diameter of the tube, m
D_h – hydraulic diameter, m
Ec – Eckert number, $u_m^2 / c_p \Delta T$
F – momentum accommodation coefficient
F_T – thermal accommodation coefficient
Gz – Graetz number, $RePrD_h/L$
h – heat transfer coefficient, W/(m^2·K)
k – thermal conductivity, W/(m·K)
$\bar{k}$ – Boltzman const., 1.38×10^{-23}J/(K·mol)
Kn – Knudsen number, λ/D_h
L – channel length, m
Nu – Nusselt number, hD_h/k
P – pressure, Pa
Pr – Prandtl number, ν/α
R – tube radius, m
r – radial coordinate, m
Re – Reynolds number, $\rho D_h u_m/\mu$
T – temperature, K
u – axial velocity, m/s
x – axial coordinate, m
v – velocity in your direction, m/s

Greek Symbols

α – thermal diffusivity, m^2/s
γ – heat capacity ratio, c_p/c_v
ζ – dimensionless x
η – dimensionless r
θ – dimensionless temperature
λ – molecular mean free path, m
μ – dynamic viscosity, kg/(m·s)
ν – kinematic viscosity, m^2/s
ρ – density, kg/ m^3

References

1. Bar-Cohen A. State of the art and trends in the thermal packaging of the electronic equipment // *ASME J. of Electronic Packaging.* 1992. Vol. 114. Pp. 257–270.
2. Kakac S., Yuncu H., Hijkata K. (eds) Cooling of electronic system // *NATO ASI Series E*, Kluwer, The Netherlands. 1992. Pp. 258.
3. Cercignani C. *The Boltzmann Equation and its Application*, Springer-Verlag, 1988.
4. Fan Q., Xue H., Shu C. DSMC simulation of gaseous flows in microchannels // *5th ASME/JSME Thermal Engineering Joint Conf., San Diego, U.S.A*, AJTE99-6519, 1999.
5. Shih J. C., Ho C., Liu J., Ta, Y. Monatomic and polyatomic gas flow through uniform microchannels // *Micro Electro Mechanical Systems (MEMS), National Heat Transfer Conference*, DSC 59. 1996. Pp. 197–203.
6. Tunc G., Bayazitoglu Y. Heat transfer in microtubes with viscous dissipation / /*Intern. J. Heat Mass Transfer.* 2001. Vol. 45. Pp. 765–773.
7. Tunc G., Bayazitoglu Y. Heat transfer for gaseous flow in microtubes with viscous heating // *Proc. of ASME Heat Transfer Division*, HTD 366-2, 2000. Pp. 209–306.
8. Tunc G., Bayazitoglu Y. Heat transfer in rectangular microchannels // *Intern. J. Heat Mass Transfer.* 2001. Vol. 45. Pp. 765–773.
9. Barron R.F, Wang X., Ameel T.A., Warrington R.O. The Graetz problem extended to slip-flow // *Intern. J. Heat Mass Transfer.* 1997. Vol. 40 (8). Pp. 1817–1823.
10. Patankar S.V. *Numerical Heat Transfer and Fluid Flow*, McGraw-Hill, New York, 1980.
11. Tao W. *Numerical Heat Transfer*, Xi’an Communication University Press, Xi’an, China, 1986.
12. Bayazitoglu Y., Kakac S. Flow regimes in microchannel single-phase gaseous fluid flow // Microscale Heat Transfer-Fundamentals and Applications, eds. S. Kakac, L.Vasiliev, Y. Bayazitoglu, and Y. Yener, Kluwer, the Netherlands, 2005.

13. Kakac S., Sun W., Yazicioglu A. A numerical study of single-phase convective heat transfer in microtubes for slip flow // *Intern. J. of Thermal Sciences.* 2007. Vol. 46 (11). Pp. 1084–1094.
14. Cetin B., Kakac S., Yuncu H. Gaseous flow in microconduits with viscous dissipation // *Intern. J. of Transport Phenomena*, 2007. Vol. 8. Pp. 297–315.
15. Cetin B., Yazicioglu A.G., Kakac S. Fluid flow in microtubes with axial conduction including rarefaction and viscous dissipation // *Intern. Comm. Heat Mass Transfer.* 2008. Vol. 35. Pp. 535–544.