PERFORMANCE CHARACTERISTICS OF A LONG HEAT PIPE

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Abstract

Analytical solution of laminar flow in long cylinder heat pipes subjected to a uniform vapor injection at evaporator section is presented. The governing partial differential equations have been transformed to a set of ordinary differential equations under some conditions. Then, the equations are solved using the integral method. Appling an appropriate polynomial for axial velocity, the conservative equations are solved for radial velocity and pressure profiles.

The results show that the trends of axial velocity distribution at different radial Reynolds number are the same and the discrepancies among them start at the middle of the channel radius where, the effect of viscose flow adjacent to the wall is much higher than the inertia effect. The results also show that the radial velocity reaches the maximum value at about 80 percent of the heat pipe radius. The maximum value is about 8 percent higher than the injection velocity.

KEYWORDS

Laminar incompressible flow, fluid flow, heat Pipe, analytical solution.

INTRODUCTION

Heat pipe is a closed device containing a liquid that transfer heat under isothermal conditions from an evaporator section to a condenser section by vaporization of liquid in the evaporator and condensation of vapor in the condenser. Liquid return flows back to the evaporator by capillary action through surface tension forces acting within a porous wick material along the inner wall of the container of the heat pipe. These two sections are separated from each other by an adiabatic section. The adiabatic section is designed to fit the external geometrical requirements of the heat pipe, i.e., spacing limitations. At the evaporator section, thermal energy is transferred from the external source to the working fluid in the heat pipe. In contrast, at the condenser section, thermal energy is transferred from the working fluid in the heat pipe to the external sink.

The advantage of using a heat pipe over other conventional methods is that large quantities of heat can be transported through a small cross-section area over a considerable distance with no additional power input to the system. Furthermore, simplicity of design and manufacturing, small end-to-end temperature drops, extremely wide temperature application rang (4 – 3000K), and the ability to control and transport high heat rates at various temperature levels are unique features of heat pipes [1-3]. Heat pipes have been applied in many ways since their introduction by Grover et al. in 1964 [4]. Some of the industrial applications of heat pipes are in boiler, furnace, dryer, heat exchanger for heat recovery [5-8], electronic equipment cooling systems [9, 10], medicine and human body temperature control [11, 12] and in the area of spacecraft cooling [13, 14].

The capillary pressure generated in a porous wick arises from surface tension forces at the vapor-liquid interface. The porous wick material with small random interconnected channel may be constructed from a wrapped screen or screen-covered grooves. The pores in the wick act as a capillary pump. The word pump is used because of the wick’s analogous role to regular pumping action on fluids in the pipes by pumps. Also, the wick serve as an effective separator between vapor and liquid phases, thereby allowing more heat to be carried over greater distance than other pipe arrangements. Along the vapor-liquid interface inside the heat pipe, the vapor temperature is related to the pressure according to the equilibrium temperature-pressure relation. Therefore, a large pressure drop in the vapor flow direction may result in a significant temperature drop along the interface, thus affecting the overall heat pipe performance. The pressure drop effect becomes more pronounced in a long heat pipe or in cases where the operating vapor pressure is relatively small, such as in liquid metal heat pipes. There have been several recent investigations [15, 16] to analyze the effects of vapor flow, particularly the effect of pressure variations on the overall heat pipe performance.

These analyses were all based on mass and momentum conservation equations and thus neglected the complex coupling with the energy equation and the thermodynamic equilibrium relation. Moreover, the boundary layer approximation was often employed [17, 18] in a problem clearly of elliptic type and the validity of this approximation in the case with application vapor pressure drop has not been assessed. In contrast to normal heat conductors, the high thermal conduction of heat pipes exists only up to a certain maximum heat flux [19, 20] which depends on several factors. One of these is the pressure drop in the vapor phase of the working fluid. This pressure drop will be analyzed for a relatively simple case, which is of some practical interest.

In the present paper the effect of vapor injection on velocity, pressure gradient distribution and the overall heat pipe performance are investigated analytically. The theoretical framework is based on the linear variation of the average axial velocity with respect to z and the thermodynamic equilibrium vapor relations. Then, the Navier-Stokes equations are solved for the study of fluid flow in a cylindrical heat pipe subjected to a uniform wall injection.

MODEL DESCRIPTION

The steady state axisymmetric motion of a laminar incompressible fluid flow with constant properties in a long cylindrical heat pipe subjected to uniform wall injection is considered (Fig. 1). It is assumed that the length of each zone, i.e., heating zone, heat shield zone and cooling zone, is large compared with the diameter and that the heating and cooling rates are constant. The vapor flow is described by the Navier-Stokes equations.

$$\frac{\partial w}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left[r v\right]=0 \tag{1}$$

$$\rho\left(w\frac{\partial w}{\partial z}+v\frac{\partial w}{\partial r}\right)=-\frac{\partial P}{\partial z}+\mu\left[\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right], \tag{2}$$

$$\rho\left(w\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial P}{\partial r}+\mu\left[\frac{\partial^2 v}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)-\frac{v}{r^2}\right], \tag{3}$$

where w and v are the velocity components in the z and r directions, respectively and P is the fluid pressure. ρ and μ are the density and viscosity of the fluid, respectively. The appropriate boundary conditions are defined as follows.

$$v(r,z)=\left|\begin{array}{lc} 0 & z=0,L \\ 0 & r=R,\ L_e<z<L_e+L_a \\ -v_{in} & r=R,\ z\le L_e \\ v_{out} & r=R,\ L_e+L_a\le z\le L \end{array}\right., \quad w(r,z)=\left|\begin{array}{lc} 0 & z=0,L \\ 0 & r=R \end{array}\right., \quad \frac{\partial w}{\partial r}(0,z)=0, \quad (4a-c)$$

where R is the inside heat pipe radius. Also, L_e, L_a, L_c are the evaporator, adiabatic and condenser length respectively and $L=L_e+L_a+L_c$.

The mass flow rate of the vapor at any cross section of the evaporator is given by:

$$2\pi Rz\rho v_{in}=\int_0^R 2\pi\rho r w dr\,. \tag{5}$$

Satisfaction of this relationship leads to:

$$w(r,z)=zg(r)\,. \tag{6}$$

Introducing the function $w(z, r)$ into the mass equation and applying the boundary condition $v(0,\ r) = 0$, the velocity component in the r-direction can be easily deduced as follows:

$$v = f(r). \tag{7}$$

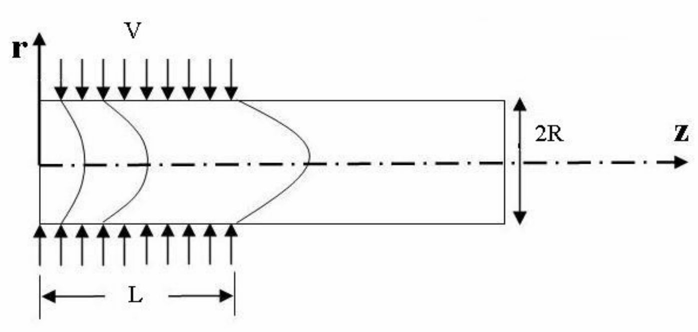

Fig. 1. Heat pipe subjected to a wall injection with coordinate system

Defining the following non-dimensional variables, the non-dimensional form of the governing equations are:

$$\frac{\partial w^*}{\partial z^*} + \frac{1}{r^*}\frac{\partial}{\partial r^*}\left[r^* v^*\right] = 0, \tag{8}$$

$$w^* \frac{\partial w^*}{\partial z^*} + v^* \frac{\partial w^*}{\partial r^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{Re}\left[\frac{\partial^2 w^*}{\partial z^{*2}} + \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial w^*}{\partial r^*}\right)\right], \tag{9}$$

$$w^* \frac{\partial v^*}{\partial z^*} + v^* \frac{\partial v^*}{\partial r^*} = -\frac{\partial P^*}{\partial r^*} + \frac{1}{Re}\left[\frac{\partial^2 v^*}{\partial z^{*2}} + \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial v^*}{\partial r^*}\right) - \frac{v^*}{r^{*2}}\right], \tag{10}$$

$$z^* = \int_0^1 r^* w^* dr^*, \tag{11}$$

$$w^* = z^* G(r^*), \tag{12}$$

$$v^* = F(r^*), \tag{13}$$

$$z^* = \frac{z}{R}, \qquad r^* = \frac{r}{R}, \qquad w^* = \frac{w}{v_{in}}, \qquad v^* = \frac{v}{v_{in}}, \qquad p^* = \frac{p}{\rho v_{in}^2}, \qquad Re = \frac{\rho v_{in} R}{\mu}$$

Substituting the above variables into eqs. (4a-e), the boundary conditions in non-dimensional form change to:

$$w^*(1,\ z^*) = \frac{\partial w^*}{\partial r^*}(0,\ z^*) = 0, \tag{14a-b}$$

$$v^*(1) = -1, \tag{14c}$$

$$v^*(0) = 0. \tag{14d}$$

EVAPORATOR ZONE

Due to axisymmetry of the vapor flow through the heat pipe, a fourth power polynomial of the radius is approximated for the velocity profile in the z-direction.

$$w_e^* = z^*(ar^{*4} + br^{*2} + 1). \tag{15}$$

Introducing eq. (15) into eq. (11) and applying the boundary condition $w_e^*(1,z^*)=0$, b is obtained in favor of 'a' and the final result will be:

$$w_e^* = z^*(1-r^{*2})[4+a(\frac{1}{3}-r^{*2})]. \tag{16}$$

Substituting equation above into eq. (8) and applying the boundary condition $v_e^*(1)=-1$ reads.

$$v_e^* = -\frac{a}{6}r^*(1-r^{*2})^2 - r^*(2-r^{*2}). \tag{17}$$

The value of 'a' is obtained by integrating eq. (10) after inserting equation above and assuming that the radial pressure gradient does not vary across the heat pipe. Then it reads.

$$a = \frac{3}{2}(\mathrm{Re}-8). \tag{18}$$

Now the axial pressure gradient is evaluated from eq. (9) at the center of the heat pipe as an average value across the pipe.

$$-\frac{1}{z^*}\frac{dp_e^*}{dz^*} = \frac{\mathrm{Re}^2}{4} - \frac{48}{\mathrm{Re}} + 8. \tag{19}$$

ADIABATIC ZONE

It is assumed that the vapor velocity would be a fully developed flow at the beginning of the adiabatic section where the injection at the wall ends, $z=L_e$. In this case, the radial velocity would be zero but, the axial velocity and pressure gradient are obtained as follows.

$$w_a^* = L_e^*(1-r^{*2})[4+a(\frac{1}{3}-r^{*2})], \tag{20}$$

$$-\frac{1}{L_e^*}\frac{dp_a^*}{dz^*} = \frac{\mathrm{Re}^2}{4} - \frac{48}{\mathrm{Re}} + 8. \tag{21}$$

CONDENSER ZONE

In this section, in contrast to the evaporation zone, the vapor flow is rejected by action of condensation process at the wall; thus, the axial and radial velocity components in the condenser will be as:

$$w_c^* = \frac{L_e^*}{L_c^*}(L^*-z^*)(1-r^{*2})[4+a(\frac{1}{3}-r^{*2})], \tag{22}$$

$$v_c^* = \frac{L_e^*}{L_c^*}[\frac{a}{6}r^*(1-r^{*2})^2 + r^*(2-r^{*2})], \tag{23}$$

$$-\frac{dp^*}{dz^*} = \frac{L_e^*}{L_c^*}(L-z^*)[-\frac{L_e^*}{L_c^*}(4+\frac{a}{3}) + \frac{16}{\mathrm{Re}}(1+\frac{a}{3})], \tag{24}$$

where

$$a = \frac{3}{2}(\frac{L_e^*}{L_c^*}\mathrm{Re}+8). \tag{25}$$

RESULTS AND DISCUSSION

A saturated water vapor flow is assumed in a cylindrical heat pipe subjected to a uniform injection at T_{sat}=350 K. The micro channel radius is $R=2\,\mathrm{cm}$ with injection length of $L=20\,\mathrm{cm}$. It can easily be shown that the maximum Mach number at $z=L$ for the axial Reynolds number, $\rho\overline{w}D/\mu=2300$, is very smaller than 0.3. Therefore, the vapor can be assumed to be incompressible.

Fig. 3 illustrates the distribution of the axial velocity at any across section of the channel flow at three different radial Reynolds numbers. To provide a generalized velocity distribution across the channel flow which is being independent of the location, the axial non-dimensional velocity is divided by the axial non-dimensional coordinate system. It can be seen that as the radial Reynolds number increases the model works well and shows the evolution of the velocity profiles reasonably. It is worth noting that the discrepancies among the curves start at about $r^* = 0.5$ where, the effect of viscose flow adjacent to the wall is much higher than the inertia effect.

Fig. 3 depicts the radial velocity against the heat pipe radius for three different radial Reynolds numbers in the evaporator section. The trends of the curves are similar and they reach a maximum value of about 1.08 at $r^* = 0.8$.

Fig. 4 reports the pressure gradient versus the radial Reynolds number in the evaporator section. The gradient increases rapidly for very small radial Reynolds numbers and approaches a constant value for large ones. It is important to note that the range of the radial Reynolds number in which the pressure gradient has large values is less than 0.5 correspond to the axial Reynolds number about 57.

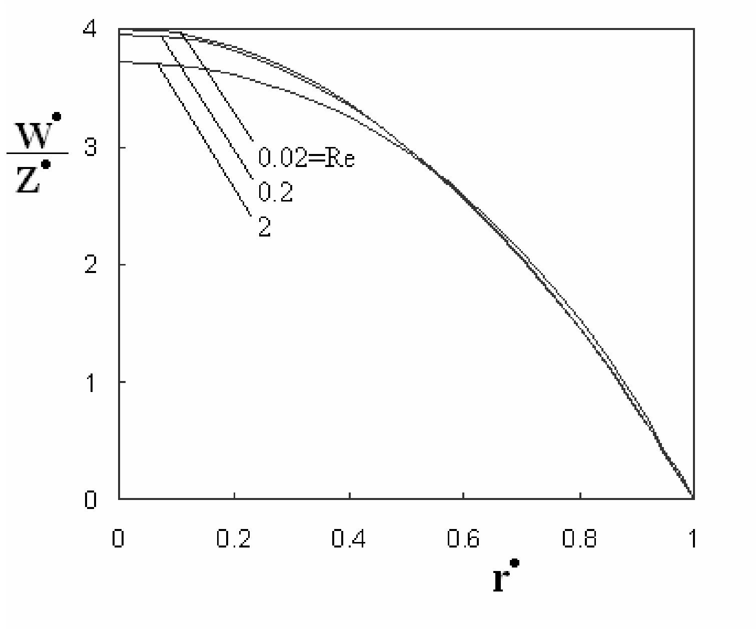

Fig. 2. The average axial velocity v.s the channel radius

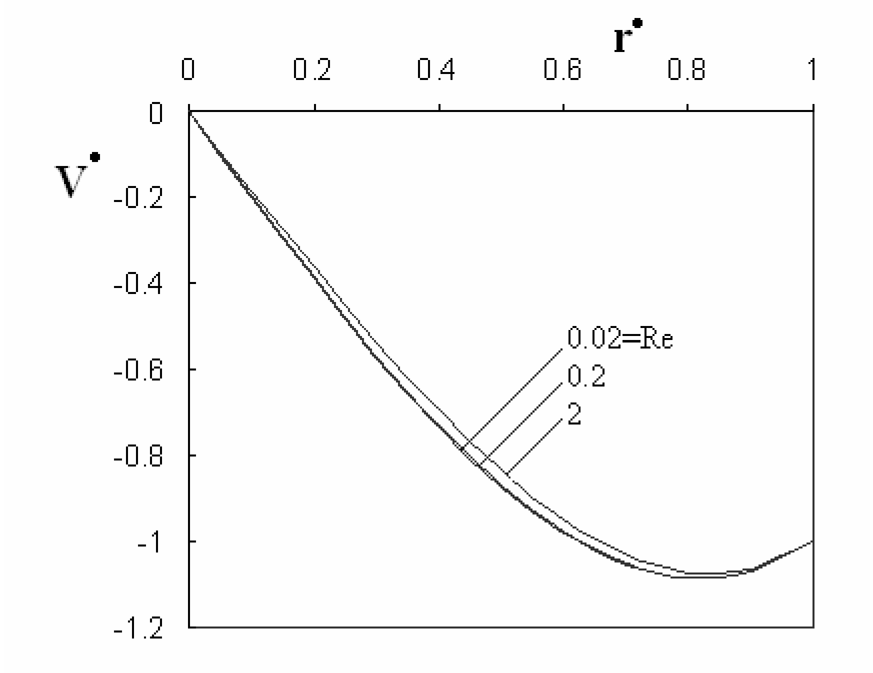

Fig. 3. The radial velocity v.s the channel radius

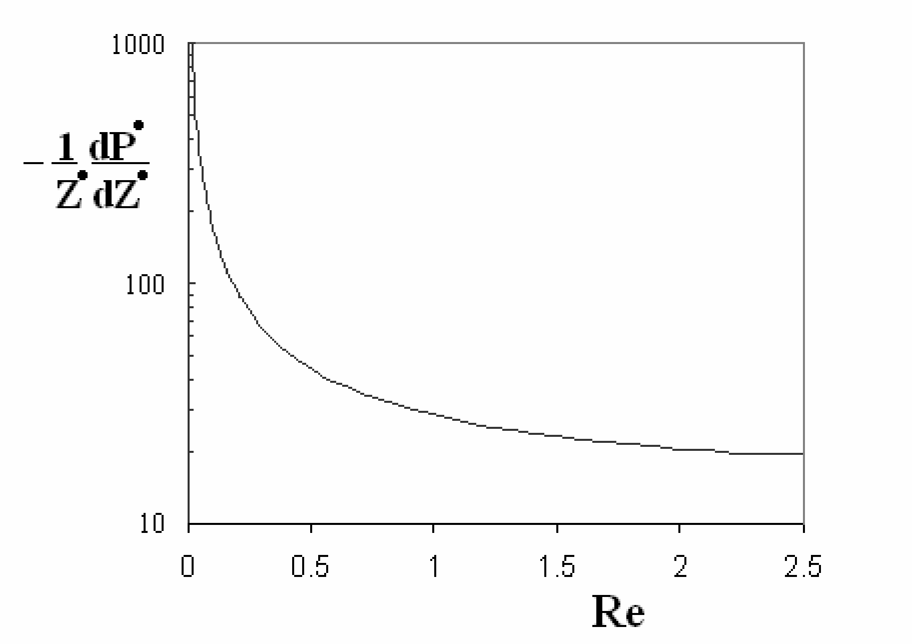

Fig. 4. The axial pressure gradient v.s the radial Reynolds number

CONCLUSIONS

Analytical analysis of fluid flow in a long cylindrical heat pipe subjected to a uniform wall injection at various radial Reynolds numbers is presented. The governing partial differential equations have been transformed to a set of ordinary differential equations under some conditions. Then, the equations are solved using the integral method.

The results show that radial velocity after injection reaches its maximum value at about 80 percent of channel radius and after that it decreases gradually to zero at the center. The trends of axial velocity distribution at different radial Reynolds number are the same and the discrepancies among them start at the middle of the channel radius where, the effect of viscose flow adjacent to the wall is much higher than the inertia effect.

NOMENCLATURE

a constant
D diameter
F function of r^*
G function of r^*
L length
P pressure
r r-direction
R radius
Re radial Reynolds number, $\rho R v_{in} / \mu$
v radial velocity
w axial velocity
z z-direction

Greek Letters

ρ dencity
μ viscosity

Superscrips

$*$ non-dimensional

Subscripts

in injection

a adiabatic

c condenser

e evaporator

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