MIXED CONVECTION IN A HORIZONTAL RECTANGULAR DUCT HEATED FROM BELOW

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Abstract

This paper deals with a mixed convection flow in a horizontal rectangular duct of transversal aspect ratio close to two, uniformly heated from below and thermally insulated elsewhere. The working fluid is water (Pr=6) and the Reynolds number associated with the main flow is small (Re=20). The heat flux supplied to the wall induces a secondary flow which manifests itself through natural convective rolls. This study has been conducted simultaneously via two ways: an experimental approach which enables us to investigate a large range of the two main control paremeters (Reynolds and Rayleigh numbers) and a numerical analysis which permits us to obtain detailed local and global descriptions of the fluid flow and heat transfer throughout the whole computational domain.

KEYWORDS

Horizontal duct, forced flow, thermal flow, mixed convection, convective rolls, incompressible fluid flow, direct numerical simulation.

INTRODUCTION

Mixed convection flow in horizontal ducts has been widely studied [1, 2, 3, 4, 5, 6]. During the first half of the twentieth century, research on this subject attempted to explain certain meteorological phenomena. More recently applications have been concerned with technological processes such as the cooling of electronic components or the production of thin films in CVD reactors; these works have mainly focused on the heat transfer enhancement related to thermoconvective structures in the flow. Results of the linear hydrodynamic stability theory have shown that the thermally stratified Poiseuille flow remains stable as long as the Rayleigh number Ra does not exceed a certain critical value, i.e. Ra*=1708. Beyond this value, the basic flow becomes unstable and two kinds of thermoconvective structures, called "transversal rolls" and "longitudinal rolls", may appear. Such results concern the case of constant temperatures imposed at the upper and lower horizontal faces of the channel; of course the upper part is colder than the lower one. The aim of our study is to investigate thermoconvective instabilities in the case of an imposed heat flux at the lower horizontal wall. This work was undertaken with a combined approach linking experiments and numerical simulations. It presents a thermal instability occurring in a mixed convection phenomenon concerning an incompressible flow in a horizontal rectangular duct heated from below. The heat flux supplied to the liquid induces a secondary flow, which is superimposed on the main forced flow.

EXPERIMENTAL SET-UP

Our experimental device is a horizontal rectangular duct uniformly heated from below [7]. Its cross section is 14 mm high, 26 mm wide, its total length is 2 m long and the thickness of its walls, made of plexiglass, is 3 mm. The entrance zone of the channel is designed to enable a fully developed Poiseuille flow at the inlet of the test zone. Moreover, the inlet temperature of the fluid is maintained constant and equal to 20°C thanks to an external heat exchanger. Next to this section, the central testing zone is heated from below over a length of 0.56 m through the lower wall made of copper. Heating is produced by Joules effect, supplying an uniform heat flux which can be adjusted from 76 $W.m^{-2}$ to 30 $kW.m^{-2}$.

The fluid flow rate is measured with a flow meter. For a given fluid velocity and heat flux, we measure fluid temperature in various locations and heights by means of thermocouples and fluid flow

velocity by Particule Imaging Velocimetry (PIV). Several series of experiments have been conducted for various flow rates and heat fluxes supplied to the wall. However, in this paper we emphasize on a particular configuration concerning a low Reynolds number (Re=20) and moderate heat flux (φ=1.5 $kW.m^{-2}$).

NUMERICAL ANALYSIS

In parallel to these experiments, computations have been conducted by means of direct numerical solution of the governing equations.

Governing equations

The problem under consideration is governed by the coupled incompressible Navier-Stokes and energy equations. However, since we assume the Boussinesq approximation stands, it implies to numerically consider only moderate heating configurations [8]. Introducing the dimensionless variables of space, velocity, time and temperature, built respectively on the height of the channel (h), the natural convection velocity ($v_{ref}=(g\beta\Delta Th)^{1/2}$), the natural convection time scale (h/v_{ref}) and the temperature difference (ΔT), the conservation equations read:

<u>Energy conservation equation in the water flow:</u>

$$\sqrt{Gr}\,.Pr.\left[\frac{\partial\theta}{\partial t}+\left(\vec{v}.\vec{\nabla}\right)\theta\right]=\vec{\nabla}^2\theta \qquad (1)$$

Associated with the following thermal boundary conditions:

- At the inlet (x=0): $\theta = 0$;
- At the lower horizontal wall (z=0):

  for $0 < x/h < L_e$: $\frac{\partial\theta}{\partial n} = 0$;

  for $L_e \leq x/h \leq L$: either $\theta = 1$ or $\frac{\partial\theta}{\partial n} = \bar{\varphi}_{adim}$;
- At the three other lateral walls: $\frac{\partial\theta}{\partial n} = 0$;
- At the outlet of the computational domain (x/h=L): open or free boundary condition [Papanastasiou et al., 1992].

Initial condition: uniform temperature field $\theta(x,y,z,0) = 0$.

<u>Incompressible fluid flow in the liquid domain:</u>

$$\vec{\nabla}.\vec{v} = 0\ ;\quad \sqrt{Gr}\left[\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}.\vec{\nabla}\right)\vec{v}\right] = -\vec{\nabla}p+\vec{\nabla}^2\vec{v}+\sqrt{Gr}\ \theta\ \vec{z} \qquad (2)$$

Associated with the following fluid flow boundary conditions:

- At the inlet: $\vec{v}(x=0,y,z) = Re\ .\vec{v}^{adim}_{Poiseuille}$ [9];
- At the lateral walls: $\vec{v} = \vec{0}$;
- At the outlet of the computational domain (x/h=L): open or free boundary condition [10].

<u>Initial condition:</u> uniform velocity field $\vec{v}(x,y,z,0) = \vec{0}$.

Numerical model

The 3D numerical model we have developed to solve the coupled fluid flow and heat transfer problem is based on a segregated approach to build up separate integral forms associated with the incompressible fluid flow problem on one hand, and the heat transfer problem on the other hand [11]. The fluid flow problem is written in a primary variable formulation and is solved using an

unconditionally stable projection algorithm [12]. Then, the spatial discretisation of the three separate integral forms is achieved following the standard finite element method, using tri-quadratic hexahedral finite elements for the velocity and temperature variables, whereas tri-linear approximation is used for the pressure. The time integration is performed with the classical first order backward Euler scheme, so at each time step the three algebraic systems related to the momentum, incompressible projection and energy conservation are solved with an iterative solver (Bi-Conjugate Gradient Stabilized, pre-conditionned with Additive Shwartz Method) provided in the Pestc toolkit [13]. This implementation enables us efficiently running high performance massively parallel computers (IBM SP4).

The modeled mixed convection configuration is characterized by an imposed water flow of Poiseuille type in a horizontal duct of rectangular cross section (transversal aspect ratio A=l/h=1.9) heated from below at a constant heat flux. The computational domain has a total length L=30h, it contains an adiabatic entrance region which extends on L_e=5h, followed by the heated from below part which extends on the remaining length L-L_e=25h. It is discretized with 480x31x16 tri-quadratic hexahedral finite elements, built up on 1 997 919 nodes.

RESULTS

Many experiments have been conducted for various flow rates and heat fluxes supplied to the wall. However, in this paper we emphasize on a particular configuration concerning a low Reynolds number (Re=20) and moderate heat flux (φ=1.5 kW.m^{-2}). At steady state, an original structure of the fluid flow was highlighted which consists of a combination of two longitudinal rolls embeded into one large transversal roll. To our knowledge this particular fluid flow structure has never been reported in the litterature untill now. This structure is characterized by a counter flow in the upper part of the duct (in the opposite direction of the main flow). This fact manifests itself through a transversal roll which takes place over the whole testing zone and extends even few duct heights upstream from the heating zone. In order to illustrate the fluid flow structure, Fig. 1 shows the velocity field, obtained experimentally by PIV, in the vertical longitudinal median plane in the nearby of the beginning (Fig. 1 left) and the end (Fig. 1 right) of the large transversal roll.

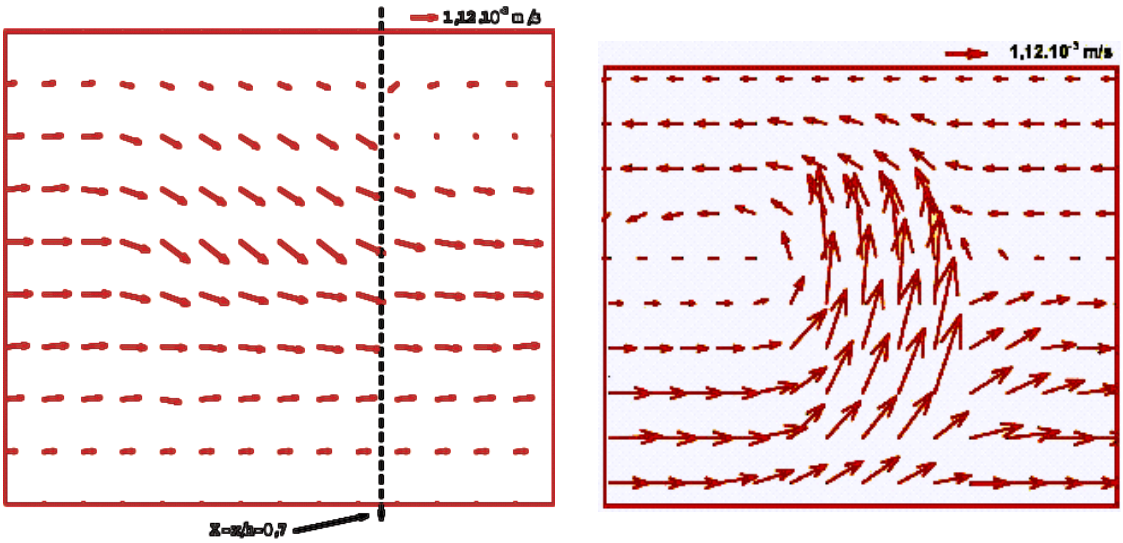

Fig.1. Velocity field in the longitudinal vertical median plane at the beginning (left) and at the end (right) of the transversal roll

In the lower part of the duct, we observe a larger value of the horizontal component of the velocity compared to the isothermal fluid flow. This results from the reduction of the effective cross section induced by the return flow in the upper part. Such result can be highlighted on the Fig. 2 where the longitudinal component of the velocity field has been plotted versus the transversal direction (Fig. 2, left) and the vertical direction (Fig. 2, right). The deviation from the Poiseuille Profile is enhanced when heat flux supplied to the wall is increased. Furthermore, the magnitude of the return flow is also increased in the upper part of the cross section.

We depict on Fig. 3 for various longitudinal coordinates, the temperature field (Fig. 3, left), the secondary velocity field (Fig. 3, center) and isovalues of the longitudinal component of the velocity field (Fig. 3, right). At the entrance zone, X=2.5 (upwind from the heated zone) only the imposed

forced flow exists, whereas as X=5 both the Poiseuille flow and the return flow coexist, the later being located in the upper part of the cross section. In the lower part, two longitudinal rolls appear, fluid is moving upward along the lateral vertical walls. At X=12.5, the thermal boundary layer in the heated zone is disturbed and two small longitudinal rolls take place symetrically around the vertical longitudinal median plane. From this point, the structure of the flow is periodically repeated downstream as displayed on Fig. 4 where the temperature field over the heated wall is plotted.

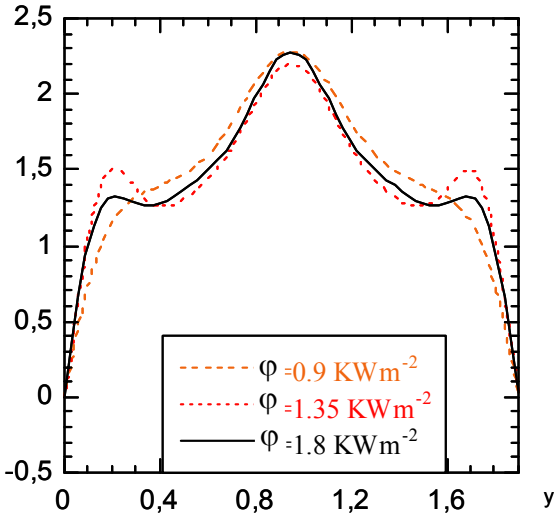

Fig. 2. Plots of the longitudinal component of the velocity field versus the transversal direction in the horozontal median plane (left) and the vertical direction in the longitudinal median plane (right)

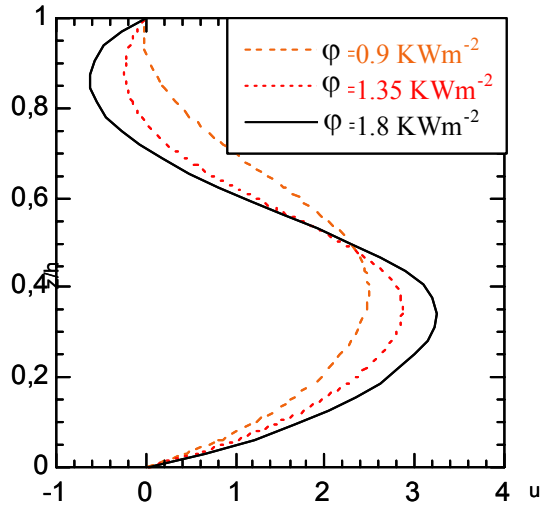

Figure 3 : Transversal thermal field (left), secondary velocity field (center) and isovalues of longitudinal component of the velocity field (right) for various longitudinal locations: a) x/h=5 ; . b) x/h=7.5 ; c) x/h=10 ; d) x/h=12.5 ; e) x/h=15 ; f) x/h=17.5 ; g) x/h=20 ; h) x/h=22.5.

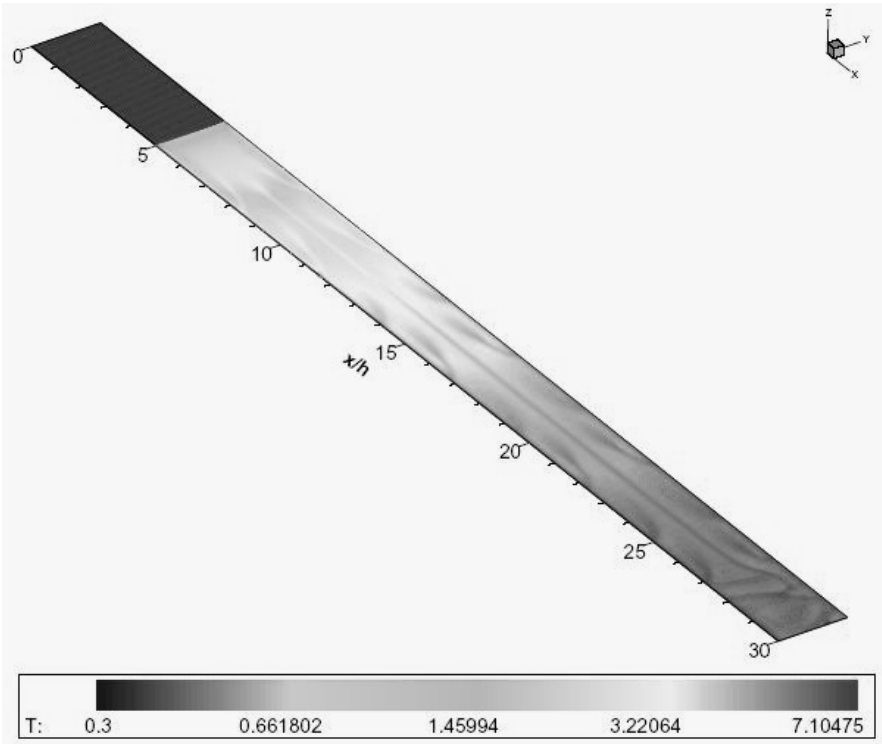

Fig. 4. Non dimensional temperature field at the heated wall [$T=(\theta-\theta_{inlet})/\Delta\theta$]

CONCLUSION

This fluid flow configuration induces a particular thermal field. In the entrance zone, when the longitudinal coordinate increases, a vertical thermal stratification takes place in the upper part of the cross section. Then, the longitudinal rolls contribute to a relatively good temperature homogeneisation in the transversal cross section.

This fluid flow structure is very interesting according to the heat transfer point of view. Indeed, secondary flow enhances significantly heat transfer at least by a factor 10. Furthermore, this situation contributes to a better homogeneization of the fluid temperature over a cross section downstream of the heating zone.

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NOMENCLATURE

$D_h=4S/P$	hydraulic diameter of the channel (S: cross-section area, P: cross-section perimeter)
g	gravity acceleration
$Gr=g\beta\Delta Th^3/\mu\kappa$	Grashoff number
h	height of the channel (14 mm)
L	length of the channel (560 mm)
n	outward unit vector normal to the domain boundary
p	dynamical pressure ($P_{hydrostatic}$: hydrostatic pressure at the outlet)
$Pr=\nu/\kappa$	Prandtl number
$Re=VD_h/\nu$	Reynolds number (V: average inlet velocity)
$\vec{V}$	non dimensional velocity vector
v_{ref}	reference velocity associated to natural convection alone
t	time
β	volumetric expansion coefficient
φ	applied heat flux at the lower horizontal surface
T_i	temperature at the inlet of the test section
ΔT	temperature difference between the heated wall and the inlet fluid (T_i)
κ	thermal diffusivity of the liquid
μ	dynamic viscosity of the liquid
ν	kinematic viscosity of the liquid
$\theta = \dfrac{T - T_i}{\Delta T}$	non-dimensional temperature

References

1. Kelly R.E. The onset and development of Rayleigh-Bénard convection in shear flow: a review, in D.B. Spaulding (Ed.) // *Physicochemical Hydrodynamics, Advance Publication*, London, 1977.
2. Abid C. and Papini F. Thermal instabilities in a mixed convection phenomenon: nonlinear dynamics // *Phys Rev E*. 1997. Vol. 56. No. 6. Pp. 6735-3744.
3. Nicolas X. Revue bibliographique sur les écoulements de Poiseuille-Rayleigh-Bénard : écoulements de convection mixte en conduites rectangulaires horizontales chauffées par le bas // *IJTS*. 2002. Vol. 41. Pp. 961-1016.
4. Yu C.H., Chang M.Y. and Lin T.F. Structures of moving transverse and mixed rolls in mixed convection of air in a horizontal plane channel // *Internat. J. Heat and Mass Transfer.* 1997. Vol. 40. Pp. 333-346.
5. Luijkx J.M., Platten J.K. and Legros J.C. On the existence of thermoconvective rolls transverse to a superimposed mean Poiseuille flow // *Internat. J. Heat and Mass Transfer.* 1981. Vol. 24. Pp. 803-817.
6. Ouazzani M.T., Platten J.K., Mojtabi A. Intermittent patterns in mixed convection // *Appl. Sci. Res.* 1993. Vol. 51. Pp. 677-685.

7. Bonnefoi C., Abid C., Médale M., Papini F. Poiseuille-Bénard instability in a horizontal rectangular duct water flow // *IJTS*. 2004. Vol. 43. Pp. 791-796.
8. Gray D.D. and Giorgini A.The validity of the Boussinesq approximation for liquids and gases // *Int. J. Heat and Mass Transfer*. 1976. Vol. 19. Pp. 545-551.
9. Nicolas X., Luijkx J.M., Platten J.K. Linear stability of mixed convection flows in horizontal rectangular channels of finite transversal extension heated from below // *Int. J. Heat and Mass Transfer* . 2000. Vol. 43. Pp. 589-610.
10. Papanastasiou T.C., Malamataris N., Ellwood K. A new outflow boundary condition // *Int. J. Num. Meth. Fluids*. 1992. Vol. 14. Pp. 587-608.
11. Medale M. and Cerisier P. Numerical Simulation of Bénard-Marangoni convection in small aspect ratio containers // *Num. Heat. Trans.*2002. *A* 42. Pp. 55-72.
12. Guermond J. L. and Quartapelle L. Calculation of incompressible viscous flows by an unconditionally stable projection FEM // *J. Comp. Physics*. 1997. Vol. 132. Pp. 12-33.
13. Balay S., Gropp W., McInnes L. C., Smith B.: PETSc 2.0.21 Users Manual // *Mathematics and Computer Science Division*. 1998. Argone National Laboratory.